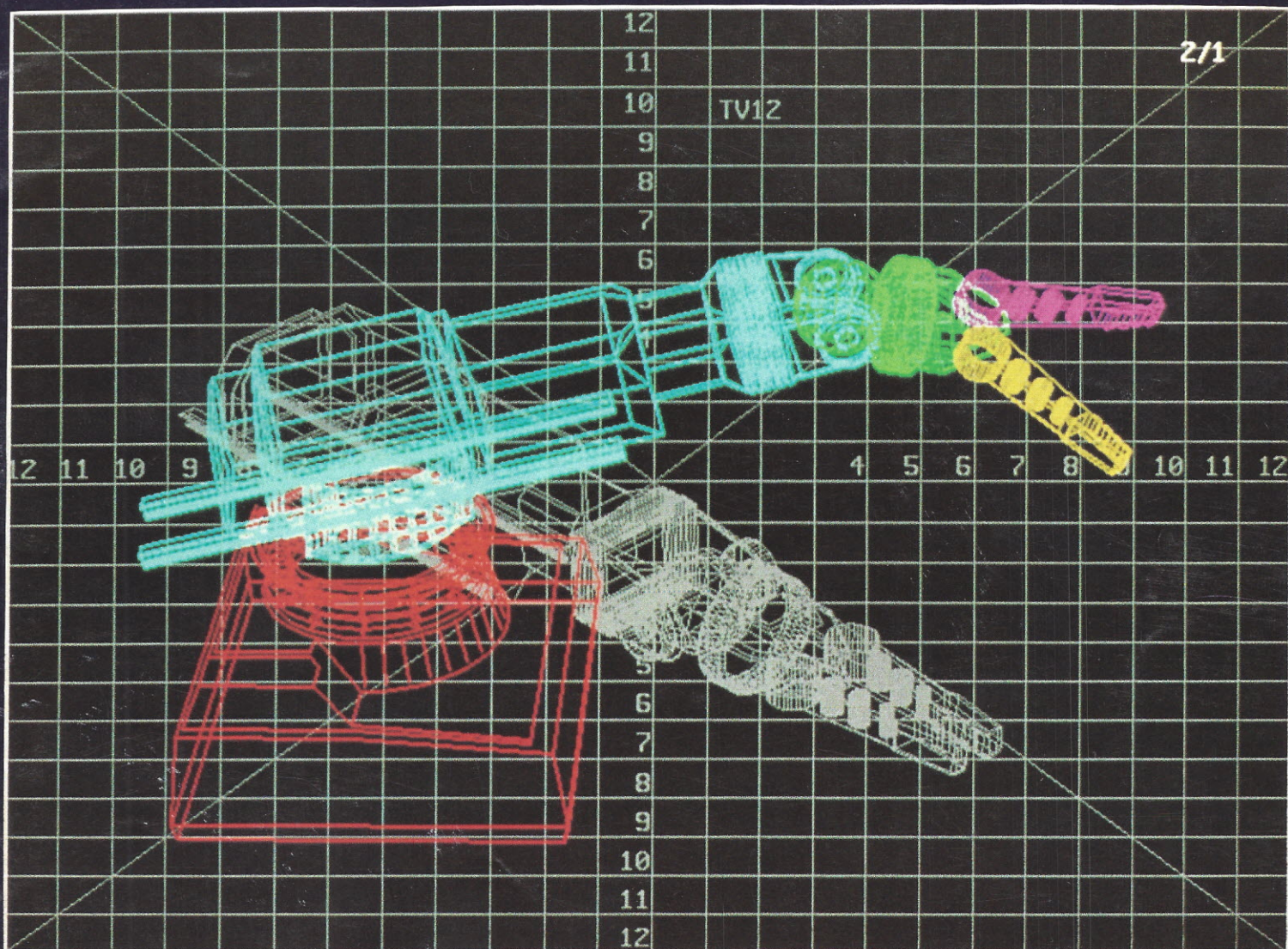


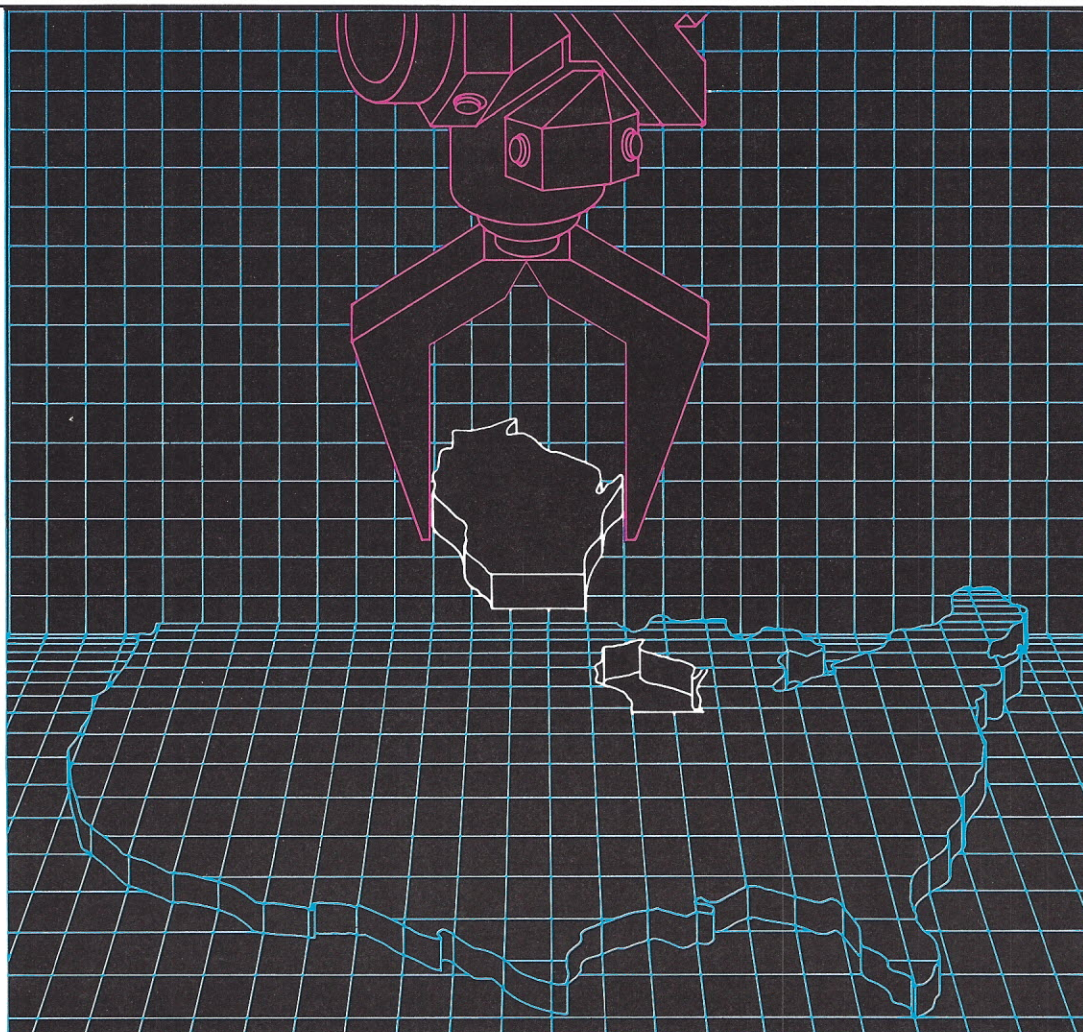
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Inverse Robot Kinematic Solutions
The Use of CAD in Robot Systems Design



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ROBOTICS

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AUGUST 1985

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About the cover: This month's cover, provided by Wavefront Technologies, is a photograph taken from the motion editing menu of a robotics simulation package. The workstation used is the Silicon Graphics Iris 2400 configured with a high-resolution color display.

Robots 9 . . . and other topics

BY CARL HELMERS

We travelled to Detroit in early June for the latest edition of Robots 9, the annual SME robotics convention. The event was held again in Cobo Hall and was attended by about 20,000 people. Exhibits and educational sessions covered the entire breadth of the industrial robotics field, and I came away with three strong impressions.

First, manufacturing cells were there in abundance. Especially fine examples were the educational and training cells from Madwed and from Prep, Inc. Also of interest were industrial cells like the General Electric assembly cell, which was to be packed up and shipped to one of that company's plants after the show.

Second, the era of the autonomous industrial/commercial robot has begun. During the convention a long-time friend dropped by our booth. A solid state physicist by training, he works for a very large computer company where his current activities include robotics-related research. He also happens to be a pioneer in private autonomous robot experimentation. He virtually dragged me over to the Intellex booth to see one of their robot arms mounted atop an autonomous mobile robot made by another firm for use in transporting semiconductor wafer carriers between clean room stations. Here was a true application of concepts that so far

have been seen only in experimental personal robots.

Third, it was at this show that GMF Robotics introduced its new robotics development language called Karel, easily the most significant new software product I saw. Karel finally brings to the real-time computer side of robotics a language and development system environment suitable for large, complex projects that must live in an organization over many, many years and through the tenure of many, many programming professionals. The language is an implementation of a Pascal-like block structured computer language together with a language-oriented development environment. Much more will be said about Karel in future issues of *Robotics Age*.

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DECODING THE ACRONYMS

Sometimes it is necessary to restate the obvious, such as to note that CIM stands for "computer integrated manufacturing." In order to accomplish CIM, we need to apply the two components of CAD (computer aided design) and CAM (computer aided manufacturing) glued together with a MAP. In today's robotics environment the MAP (manufacturing automation protocol) is becoming a standard form of hardware and software telecommunications techniques. One way to implement the CAM part of CIM is to combine one or more FMS (flexible manufacturing system) work cells, often coupled by AGVs (automatic guided vehicles) using the MAP for communications.

VISIONS OF THE FUTURE

From time to time, *Robotics Age* peers into the future. At a recent staff meeting we did this by deciding on themes for forthcoming issues and organizing them into an editorial calendar for 1986. The calendar is available to those interested in writing for us—drop a line and we will send you a copy. ■

Calendar

AUGUST

19-20 August. Developing End-of-Arm Tooling for Industrial Robots. Norcross (Atlanta), Georgia. Contact: Diane Korona, Program Administrator, Robotics International of SME, One SME Dr., PO Box 930, Dearborn, MI 48121, telephone (313) 271-1500, ext. 392.

This workshop, worth 12 professional credits toward the SME recertification program, will focus on end effectors and will cover analysis of part, process, and robot; productivity; and design techniques. Also to be addressed are type of power to be used, floor layout and work envelope size, part configuration, cycle time, and robot characteristics. The conference will be held again on 22-23 October at the Boston Park Plaza Hotel, Boston, MA.

21-22 August. Applying Sensors in Robot Applications. Norcross (Atlanta), GA. Contact: Diane Korona, Program Administrator, Robotics International of SME, One SME Dr., PO Box 930, Dearborn, MI 48121, telephone (313) 271-1500, ext. 392.

Highlights of this course will include sensor types and functions, sensor selection techniques, interfacing sensors to system components, and designing sensors into tooling. The course is worth 12 credits toward the SME recertification program.

27-29 August. Flexible Manufacturing Cells. Americana Hotel, Dearborn, MI. Contact: Sally Davis, Special Programs Division, Computer and Automated Systems Association of SME, One SME Dr., PO Box 930, Dearborn, MI 48121, telephone (313) 271-1500, ext. 383.

This seminar will provide an introduction to and definition of flexible manufacturing cells. Planning considerations, components,

peripherals, gauging and inspection, and software considerations will also be discussed. A number of user case study presentations have been scheduled by Cummins Engine, Rockwell International, and Digital Equipment, among other companies.

SEPTEMBER

6-8 September. International Personal Robot Congress & Exposition. Moscone Center, San Francisco, CA. Contact: Sharon D. Smith, Chair, IPRC '85 Organizing Committee, 8822 S. Martin Lane, Conifer, CO 80433, telephone (303) 674-5650.

The second annual IPRC, expected by the organizers to attract over 2500 personal robot enthusiasts, will feature seminars on personal robot software and hardware, human services, robots in space, the business of personal robots, and personal robots in education.

9-10 September. Second International Conference on Advanced Robotics. Keidanren Kaikan Bldg., Tokyo, Japan. Contact: Mr. A. Yasutake, Organizing Secretary, Japan Industrial Robot Association, Kikai Shinko Kaikan Bldg., 3-5-8, Shibakoen, Minato-ku, Tokyo, 105 Japan.

The primary objective of this conference is to provide an international exchange of information on intelligent, mobile, and sensory control robots. Approximately 70 papers will be presented.

9-11 September. OEM Design '85. Philadelphia Civic Center, Philadelphia, PA. Contact: Penton Exposition Division, 122 E. 42nd St., New York, NY 10017, telephone (212) 867-9191.

Papers to be presented at this conference will include the following topics: vision systems, off-line

programs for robots, justifying robots, the status of computer-integrated manufacturing (CIM) as a business strategy, CIM in electronics manufacturing, flexible manufacturing systems, and computers as design components.

10-12 September. Midcon/85. Chicago, IL. Contact: Nancy Hogan, Electronic Conventions Management, 8110 Airport Blvd., Los Angeles, CA 90045, telephone (213) 772-2965.

The Midcon/85 conference will address current or near-term advances, trends, and applications of electronic technology and product manufacture.

11-13 September. Fifteenth International Symposium on Industrial Robots. Keidanren Kaikan Bldg., Tokyo, Japan. Contact: Mr. Y. Komori, Organizing Secretary, 15th ISIR, Japan Industrial Robot Association, Kikai Shinko Kaikan Bldg., 3-5-8, Shibakoen, Minato-ku, Tokyo, 105, Japan.

More than 100 papers will be presented on the latest research, developments, applications, and socioeconomic evaluation of industrial robots during this symposium. In addition, an international exhibition of industrial robots and their applied systems equipment will be held September 12 through 16 at the exhibition hall of the International Trade Fair Center, Harumi, Tokyo. After the symposium, a study tour during which participants may visit Japanese robot manufacturers, users, and research laboratories is also being planned.

17-25 September. EMO Hannover. Hannover Fairgrounds, Hannover, Federal Republic of Germany. Contact: EMO General Commissariat, Messegelände, D-3000 Hannover 82, Federal Republic of Germany.

The latest advances in manufacturing technology and automation

will be demonstrated in Hannover by 849 companies from 36 countries at this exhibit which will feature computerized automation, control technologies, CAD-CAM, sensor and diagnostic technology, assembly, manipulating equipment and industrial robots, and flexible manufacturing systems. Foreign visitors will receive a free season ticket at the entrances to the exhibition on presentation of their passports.

18-19 September. International Congress for Metalworking and Automation. Hannover, Federal Republic of Germany. Contact: EMO General Commissariat, Messegelände, D-3000 Hannover 82, Federal Republic of Germany.

The International Congress for Metalworking and Automation will take place during the 6th EMO in Hannover to review current technological developments in metalworking. Primary subjects to be discussed include flexible manufacturing, assembly and manipulating equipment, sensor and diagnostic equipment, and computer-assisted manufacturing.

23-25 September. Space Tech '85. Disneyland Hotel Convention Center, Anaheim, CA. Contact: Gregg Balko, Technical Activities Dept., Society of Manufacturing Engineers, One SME Dr., PO Box 930, Dearborn, MI 48121, telephone (313) 271-1500, ext. 368.

Sponsored by nine technical and engineering societies, SPACE TECH '85 will focus on the engineering technologies and solutions required to make space industrialization practical and economical. There will be sessions discussing artificial intelligence and robotics; guidance, controls, and sensors; materials processing and applications; simulation and software; composites; space structures; habitation; propulsion; and manufacturing in space.

In The Robotics Age

Edited by Stephanie vL Henkel

AT ROBOTS 9

At the beginning of June we spent an informative week at Robots 9, the Big One in industrial robotics. There were about 250 displays, some quite spectacular and some less flashy but offering unexpected little gems of ingenuity. We tried to visit every booth and we hope our readers who attended the convention paid a call on us. Herewith, our overall impressions of Robots 9—in no way a complete accounting but, we hope, enough to suggest the flavor of the show.

Most of the machinery we saw had automotive manufacturing applications, which was hardly surprising since we were in Detroit. Robots welded spots and seams, robots spray painted (simulated by ribbons streaming from the nozzles), and robots with high-pressure water jets carved out inside body parts for cars.

Smaller assembly robots were prominent as well, and one arrangement in particular caught our eye. Gulf + Western had set up a flexible manufacturing

system wherein an IBM 7535 assembled 10 and 20 amp residential circuit breakers, a Seiko PN-700 transferred the part from the IBM to an Hitachi PW10II, and the Hitachi disassembled the breaker and reloaded the parts into a feeder. The three work envelopes overlapped but there was no jostling (Photo 1).

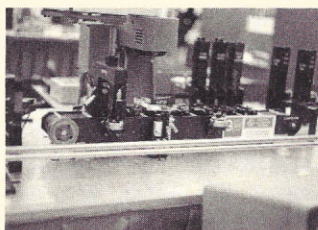


Photo 1.

We had breakfast with Bob, Jurgens/Acme's three-pronged end effector developed for small parts handling. Bob (whose creators grew so fond of him they felt he needed a name) has a payload of 1½ lbs. and a gripping force of 28 ounces. He held a succession of variously shaped wooden blocks (Photo 2) and finally a raw egg without mishap.

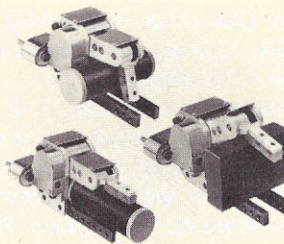


Photo 2.

We heard an ICC spokesman declare that "hardware is only something to hold the software" and stayed around to learn about the company's new integrated cell controller that can control up to 32 axes of motion and over 1000 I/O

points—from an IBM PC. The motion control is based on an advanced digital servo technology that reduces components and replaces analog circuitry with digital firmware. Sixteen of these units can be linked together, bringing the Factory of the Future pretty close.

Our eyes were what led us over to the McDonnell Douglas CAD/CAE/CAM presentation. Running on a DEC Microvax II, the program simulated, among other wonders, two robots moving simultaneously (Photo 3).

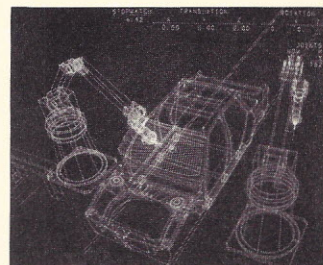


Photo 3.

The images were stunning in their color and crispness and we wondered how much time the company's design engineers spend simply enjoying their CRTs.

Next we went to admire the King Kong of robot arms, Yaskawa's Motoman® (10 ft. tall, 20 ft. diameter work envelope, 265 lbs. payload including end effector). Motoman was designed for material handling, welding, palletizing, grinding, and deburring but at the convention it was showing

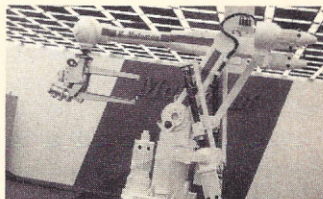
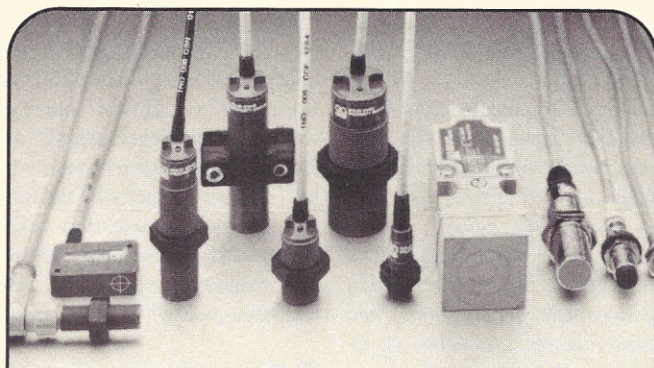


Photo 4.



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In The Robotics Age

off, balletically displaying in huge scale all its axes of motion (Photos 4,5).

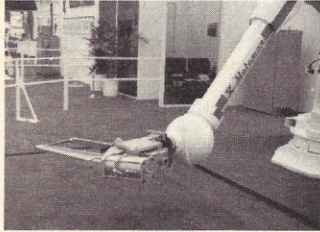


Photo 5.

No robot show would be complete without the Spine spray painting robot from Sweden (Photo 6). The design of this one-of-a-kind arm is based on the human backbone. A stack of stainless steel discs is threaded by two pairs of opposed cables powered by hydraulic actuators. The discs are free to rock back and forth

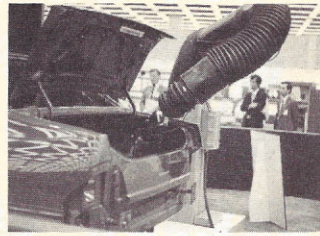


Photo 6.

about their centers in all directions, under the control of the cables. A resistive wire sensor tracks the motion of the cables, providing feedback to the control system. The mechanism is amazingly flexible and the program is accurate enough for painting cars. Spine does not foresee any other application for the robot, since even laying on sealants would require greater precision than the company cares to give this product.

South Korea's only manufacturer of industrial robots was exhibiting a robotic welding arm and we stopped by the Daewoo booth (Photo 7). A

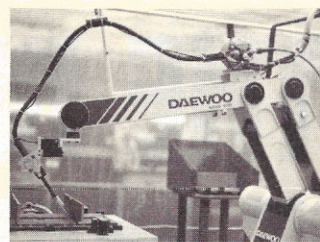


Photo 7.

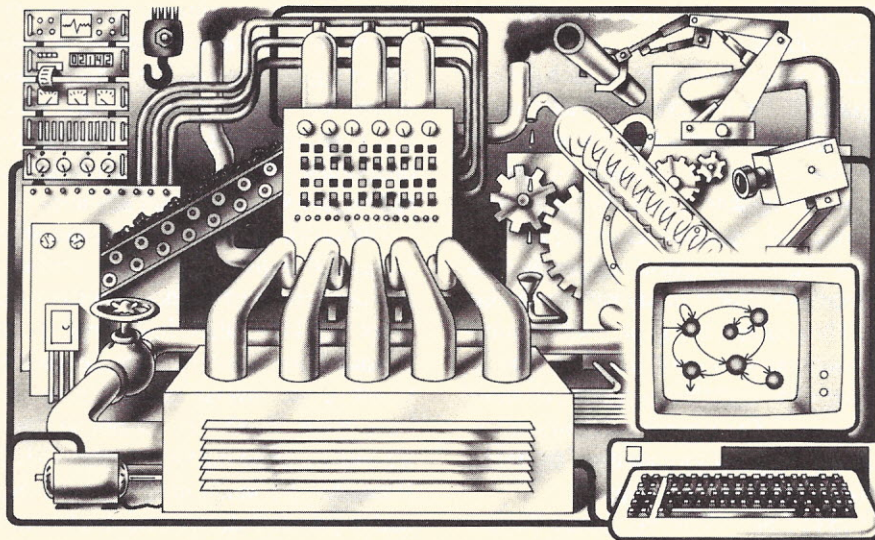
spokesman for the new company said its unchallenged position was due in part to no other Korean company's having as yet developed the drive motors, and too, until recently robotic welders were not much in demand in Korea. He anticipated that his product would end up

cost-competitive with similar welders from Japan.

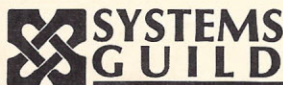
We heard a voice saying "Help me" and we hurried over to investigate. Intel was demonstrating a remarkable speech recognition system and "help me" brought up the menu. This system is so user-friendly that it takes into account human impatience. People, an Intel spokesman explained, never say the same word twice in precisely the same way. An unsophisticated voice recognition system responds to such deviations by asking "What?" over and over until the user is ready to throw it out the nearest window. Intel's system understands speech nuances rather well, and when there is a question it tells the

Continued on page 34

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THE EXACT INVERSE KINEMATICS SOLUTIONS FOR THE RHINO XR-2 ROBOT MANIPULATOR

M. Shahinpoor

Department of Mechanical Engineering
University of New Mexico
Albuquerque, NM 87131

The inverse kinematics problem in robotic manipulation is to find the vector of the joint angles, say for an n -axis revolute manipulator, given the position and the orientation (attitude) of its gripper. The attitude vector—or, more generally, the attitude matrix—describes the position of the origin, and the rotational orientation, of a convective coordinate frame embedded in the gripper frame with respect to a fixed reference frame at the base of the robot manipulator.

No exact inverse kinematics solutions exist as yet for a general n -axis robot, or, in particular, for the general 6-axis robot manipulator widely used in high-tech automation. It is sometimes possible to solve the inverse kinematics problem for certain special classes of n -axis robot manipulators, even though in most cases the amount of algebraic and trigonometric manipulations turns out to be horrendous. The mathematical problem is composed of a set of twelve equations for n unknowns pertaining to an n -axis manipulator. The problem is complex with a nonunique set of solutions or sometimes no solutions at all. The inverse kinematics is a subset of a much larger and much more difficult "the inverse dynamics." The inverse dynamics essentially seeks to find the vector of the joint torques given the forces and the torques (dynamic attitude) on the gripper.

Historically, Pieper [1] and Pieper and Roth [2] were the first to derive a set of solutions for a number of special robot manipulators. They also devoted a portion of their work to iterative-type algorithms to numerically solve the inverse kinematics problem. Paul [3] has demonstrated inverse kinematics solutions for the Stanford (cylindrical polar) and a 6-axis elbow manipulator. Hollerbach and Gideon [4] have also obtained a set of inverse kinematics solutions for a special class of robot manipulators. The practice in the industry is to consider a robot manipulator and try to obtain inverse kinematics solutions to be incorporated in the pertinent software.

Sometimes it is almost impossible to find exact solutions to the inverse kinematics problem of general 6-axis industrial robots. Attempts have been made to tackle this problem by means of iterative algorithms. For example, Lumelsky [5] presented iterative solutions for the inverse kinematics problem for one type of robot manipulator and Campbell [6] and Shahinpoor and Campbell [7] have employed Powell's [8] optimization technique to arrive at an iterative algorithm for the inverse kinematics solution of a general n -axis robot manipulator. However, the typical computation time for a 6-axis robot on a VAX-11/780 computer has been observed to be about 1 to 25 seconds which is undesirable. One should mention also the recent exact solutions of Tsai and Morgan [9] and Shahinpoor, Jamshidi, and Kim [10].

This article presents a set of exact solutions for a Rhino XR-2, a 5-axis, 6 degrees of freedom revolute robot. In the next section are developed the governing equations for the Rhino XR-2 robot arm kinematics. In deriving the governing equations the Denavit-Hartenberg [11] homogeneous transformations are used to set up the inverse kinematics problem. After the solutions have been found, the computer algorithm for the inverse kinematics solutions for the Rhino XR-2 will be presented. The Rhino XR-2 is widely used in many U.S. educational institutions and in private industry, and it is becoming a standard teaching and research tool in robotics education.

RHINO XR-2 ROBOT KINEMATICS

The Rhino XR-2 robot manipulator is an open-loop kinematic chain (Photo 1). One end of this manipulator is fixed in an inertial frame and is called the base.

At the other end of the chain is the gripper or an end-effector that is activated by a motorized four-bar mechanism. The chain is made up of a series of links and joints. The position of the

end-effector or the gripper is described by a position vector extended from the origin of the base coordinate frame to the origin of a convective frame attached to the gripper. The orientation of the gripper is described by a rotation matrix expressing the relative rotation of the gripper frame with respect to the reference frame at the base. The totality of position and orientation of one coordinate frame with respect to another can be described by a 4 by 4 matrix that has been called the attitude matrix [10]. These matrices were originally conceived of and derived in a classic work by Denavit and Hartenberg [11] and are called Denavit-Hartenberg (D-H) A matrices. The D-H matrices are essentially 4 by 4 homogeneous transformations sufficient for the description of the complete kinematic properties of lower-pair mechanisms. Lower-pair mechanisms are defined by mutual contact between one part and another, with one surface sliding with respect to the other.

Four parameters represent the coordinates of any lower pair. These parameters are a , α , θ , and d . The parameter a is known as the link length, α is the link twist, θ is the link angle, and d is the link distance. The coordinates for the Denavit-Hartenberg notation are defined in Figure 1. In general, two links are connected by a "lower-pair" joint that has two contacting or sliding surfaces. As mentioned before, a joint axis (say the i th joint) is established at the connection of two links.

The coordinates are chosen as follows:

1. The z_i coordinate is along the axis of motion or rotation of the $(i+1)$ th joint.
2. The x_i axis is in the direction of normal to both z_i and z_{i-1} axes, pointing away from the z_{i-1} axis (such as to make an r.h. coordinate with z_i and z_{i-1}).
3. The y_i coordinate is chosen so as to make the $x_i y_i z_i$ coordinate system a right-handed coordinate system.
4. The distance between the two normals a_i and a_{i-1} is called the link distance d_i .

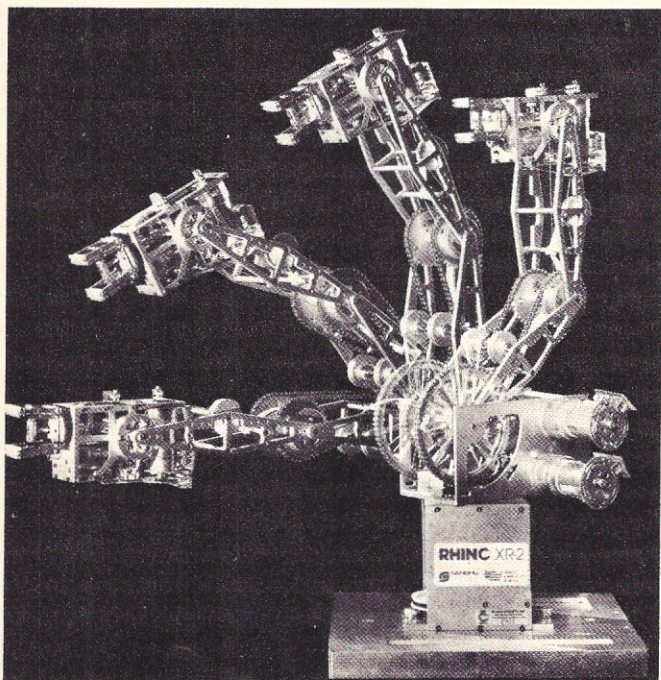


Photo 1. The Rhino XR-2 robot manipulator is shown in multiple exposures.

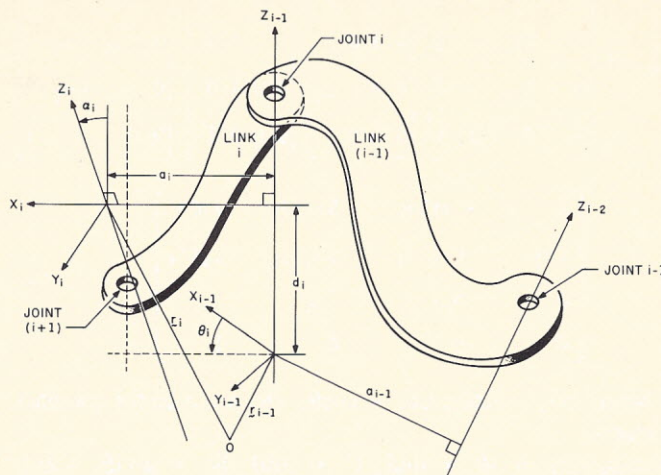


Figure 1. The coordinates for the Denavit-Hartenberg notation can be generalized as two links connected by a "lower-pair" joint that has two contacting or sliding surfaces. A joint axis is established at the connection of two links.

5. The lengths of the normal a_i is the minimum distance between the z_i and z_{i-1} axes. If this is zero, the direction of a_i becomes arbitrary. a_i is also referred to as the length of the link i .

6. The relative orientation of the x_i axis with respect to x_{i-1} axis is called $\theta_i \equiv (x_i, x_{i-1})$, or the vector of the joint angles.

7. The relative orientation of z_i axis with respect to z_{i-1} axis is called $\alpha_i \equiv (z_i, z_{i-1})$, or the vector of the twist angles.

The four parameters d_i , θ_i , a_i and α_i constitute a minimally sufficient set to determine the kinematic configuration of each link of the robotic arm. Note that

$d_i \equiv$ joint distance, $\theta_i \equiv$ joint angle, $a_i \equiv$ link length, $\alpha_i \equiv$ length twist angle.

Note that for a plane revolute joint, generally d_i , a_i and α_i are all constant, while θ_i varies as link i rotates about the axis of joint i . On the other hand, for a prismatic joint θ_i , a_i and α_i are constant while d_i varies as link i slides along the axis of joint i .

Once the D-H coordinate system for each link is established, a homogeneous transformation matrix can easily be developed relating the i th coordinate frame to the $(i-1)$ th coordinate frame. Referring to Figure 1, it is clear that a point $\underline{r}_i$ expressed in the i th coordinate system may be expressed in the $(i-1)$ th coordinate system as $\underline{r}_{i-1}$ by performing the following successive transformations:

1. Rotate about the z_{i-1} axis by an angle θ_i to align the x_{i-1} and x_i axis, $R(z_{i-1}, \theta_i)$,
2. Translate along the z_{i-1} axis a distance d_i to bring the x_{i-1} and x_i axes into coincidence; $T(z_{i-1}, d_i)$,
3. Translate along the x_i axis a distance a_i to bring the two origins into coincidence; $T(x_i, a_i)$,
4. Rotate about the x_i axis an angle α_i to bring the two coordinate systems to completely coincide; $R(x_i, \alpha_i)$.

Thus the complete composite transformation of link i with respect to link $i-1$ or joint i with respect to joint $i-1$ is

(1)

$$H_{i-1}^i = T_{i-1}^i = A_{i-1}^i = R(z_{i-1}, \theta_i) T(z_{i-1}, d_i) T(x_i, a_i) R(x_i, \alpha_i).$$

$$\text{or } A_{i-1}^i = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

where $S\theta_i = \sin\theta_i$, $C\theta_i = \cos\theta_i$, etc. are used for compact notation.

Similarly, $S_i = S\theta_i = \sin\theta_i$, $C_i = \cos\theta_i$, $S_{ij} = \sin(\theta_i + \theta_j)$, $C_{ijk} = \cos(\theta_i + \theta_j + \theta_k)$, etc.

We frequently omit the subscript $i-1$ and call A_{i-1}^i simply A_i matrices pertaining to the i th joint or link. Note that A_i has generally a form:

$$A_i^i = A_i = \begin{bmatrix} n_{x_i} & o_{x_i} & a_{x_i} & p_{x_i} \\ n_{y_i} & o_{y_i} & a_{y_i} & p_{y_i} \\ n_{z_i} & o_{z_i} & a_{z_i} & p_{z_i} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3)$$

and thus its inverse is simply A_i^{-1} such that

$$A_i^{i-1} = A_i^{-1} = \begin{bmatrix} n_{x_i} & n_{y_i} & n_{z_i} & -(p \cdot n) \\ o_{x_i} & o_{y_i} & o_{z_i} & -(p \cdot o) \\ a_{x_i} & a_{y_i} & a_{z_i} & -(p \cdot a) \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4)$$

Furthermore if $\underline{r}_i$ is the homogeneous coordinate of i th joint and $\underline{r}_{i-1}$ is the homogeneous coordinate of the $(i-1)$ th joint, it is clear that

$$\underline{r}_i = A_{i-1}^i \underline{r}_{i-1}, \quad (5)$$

where

$$\underline{r}_i = (x_i, y_i, z_i, 1)^T, \quad (6)$$

$$\underline{r}_{i-1} = (x_{i-1}, y_{i-1}, z_{i-1}, 1)^T. \quad (7)$$

The following algorithm can be used to establish the link coordinate system:

Algorithm 1. This procedure completely assigns link coordinate systems compatible with D-H transformations to a given n degree of freedom robot arm. Adjacent links can be related to each other kinematically through a set of 4 by 4 homogeneous transformation matrices. The first coordinate system pertaining to the base is denoted by a homogeneous system $(x_0, y_0,$

$z_0, 1)^T$ at the supporting base. The origin of this system is denoted as the zeroth joint.

1. Establish the base coordinate (x_0, y_0, z_0) such that z_0 axis lies along the axis of motion of joint 1.
2. Initialize and loop for each $i = 1, 2, \dots, n$.
3. Align all z_i with the axis of motion or rotation of the joint $i + 1$.
4. Establish the origin of the i th coordinate system either at the intersection of z_i and z_{i-1} axes and the z_i axis or at the intersection of common normal between z_i and z_{i-1} axes and the z_i axis.
5. Establish the x_i axis at each i th joint, either by $\underline{x}_i = \pm(z_{i-1} \times z_i)$ or along the common normal between the z_{i-1} and z_i when they are parallel.
6. Establish the y_i axis by $\underline{y}_i = (z_i \times \underline{x}_i)$ to complete the right-handed coordinate system.
7. Find d_i as the distance from the origin of the $(i-1)$ th coordinate system to the intersection of the z_{i-1} axis and x_i axis.
8. Find a_i as the distance from the intersection of the z_{i-1} axis and the x_i axis to the origin of the i th coordinate system.
9. Find θ_i as the angle of rotation from the x_{i-1} axis to the x_i axis; $\theta_i \equiv (x_{i-1}, x_i)$.
10. Find α_i as the angle of rotation from the z_{i-1} axis to the z_i axis about the x_i axis; $\alpha_i \equiv (z_{i-1}, z_i)$.

SPECIFICATION OF nT_6 MATRICES

The description of the end of the manipulator, in terms of homogeneous transformations, is given by

$${}^nT_6 = A_n A_{n+1} \dots A_6, \quad (8)$$

with respect to link coordinate frame $n-1$. The description of the end point of any link i with respect to link coordinate frame j is

$${}^jT_i = A_{j+1} A_{j+2} \dots A_i, \quad j < i, \quad (9)$$

Commonly the end of the manipulator, link coordinate frame 6, with respect to the base coordinate is given by the following transformation

$$T_6 = A_1 A_2 A_3 A_4 A_5 A_6. \quad (10)$$

This is the backbone of the inverse kinematics problem for a general 6-axis robot.

Note that for a revolute joint, all of the parameters are held constant except for the link angle θ . For a prismatic joint the only variable is the distance d , while the other parameters are held constant. The four link parameters present sufficient information to mathematically represent the lower pair. Once the Denavit-Hartenberg parameters are obtained, a homogeneous transformation matrix can be developed that will transform one link coordinate frame with respect to the next.

Thus, each A matrix provides a means of transforming from the end of one link to the beginning of another. Matrix A would represent a transformation from the base coordinate vector (usually $[0, 0, 0, 0, 0, 0]^T$), to the end of the first link. By performing consecutive matrix multiplication operations on the A matrices, it is possible to transform from the base coordinate vector to the end effector vector (also called the end point state attitude vector). For an n -axis manipulator this would be expressed by the matrix operation:

$$T_n = A_1 A_2 A_3 A_4 A_5 A_6 \dots A_n.$$

This means that if the Denavit-Hartenberg parameters and the joint coordinate vector is known, then the direct kinematic solution can be obtained for a general n-link manipulator. The matrix product of the individual A matrices is referred to as T_n . This is the transformation from base to end effector and expresses the position and rotation of the end effector with respect to the base.

We will now apply the above results for obtaining the exact solution to a Rhino XR-2 robot manipulator.

GOVERNING EQUATIONS FOR THE EXACT INVERSE KINEMATICS SOLUTIONS FOR THE RHINO XR-2 ROBOT

Before attempting to find the exact inverse kinematics solutions for the Rhino XR-2, we should mention that Paul [3] has outlined a procedure that can be used for generating a closed-form solution to some classes of manipulators. Basically, it involves trigonometric manipulation and matrix regrouping to ensure element-to-element equality within the matrices T_n and the attitude vector. One should also mention the work of Pieper and Roth [2] in relating the analysis of closed-loop and open-loop mechanisms as well as Duffy and Rooney [12] and Duffy [13], who have applied the sine and cosine laws of spatial triangles to the equivalent spatial mechanism of a manipulator and have obtained closed-form solutions of many special manipulators. In our attempt to obtain four sets of closed-form solutions to a general 6-axis robot manipulator, we are guided by these findings.

Consider a schematic of the Rhino XR-2 robot manipulator as depicted in Figure 2a and b.

Based on Figure 2a and b and Algorithm 1, the following general table for joint parameters can be constructed:

TABLE 1
General Table for Joint Parameters of the Rhino XR-2 Robot Manipulator

Joint	θ_i	α_i	a_i	d_i
1	θ_1	-90°	0	d_1
2	θ_2	0°	a_2	0
3	θ_3	0°	a_3	0
4	θ_4	-90°	a_4	0
5	θ_5	0°	0	d_5

The D-H attitude matrices can now be derived based on equation 2 and Table 1 such that

$$A_0^1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (11)$$

$$A_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (12)$$

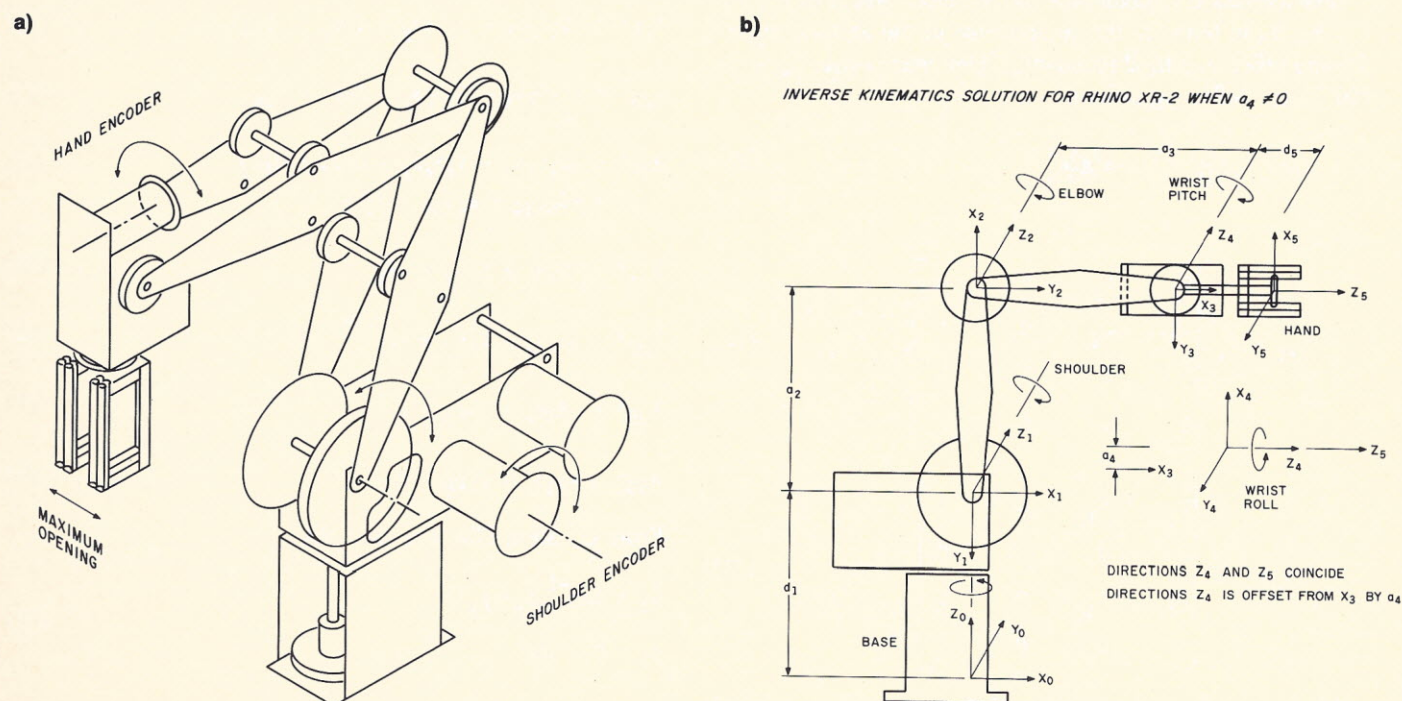


Figure 2. A schematic drawing of the Rhino XR-2 (Figure 2a) shows the configuration of the arm, the end effector, and the shoulder and hand encoders. Kinematics solutions for the robot's axes of motion are represented in Figure 2b.

$$A_2^3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3 C_3 \\ S_3 & C_3 & 0 & a_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (13)$$

$$A_3^4 = \begin{bmatrix} C_4 & 0 & -S_4 & a_4 C_4 \\ S_4 & 0 & C_4 & a_4 S_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (14)$$

$$A_4^5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (15)$$

Now the inverse kinematics equations read

$$T_0^5 = A_0^1 A_1^2 A_2^3 A_3^4 A_4^5, \quad (16)$$

where T_0^5 matrix is known and is given by

$$T_0^5 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (17)$$

Now the task is to determine the five unknowns, i.e., $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ in terms of the components of the attitude matrix T_0^5 and other structural parameters. One may rewrite equation (16) in the following fashion

$$(A_1^2)^{-1} (A_0^1)^{-1} T_0^5 = A_2^3 A_3^4 A_4^5, \quad (18)$$

where

$$(A_1^2)^{-1} = \begin{bmatrix} C_2 & S_2 & 0 & -a_2 \\ -S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (19)$$

$$(A_0^1)^{-1} = \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & -1 & d_1 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (20)$$

Now one can expand equation (18) to arrive at a set of 12 equations for the five unknowns $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$. Obviously there may exist multiple solutions or no solutions at all. Let A^* be a 4 by 4 matrix defined such that

$$A^* = A_2^3 A_3^4 A_4^5 = (A_1^2)^{-1} (A_0^1)^{-1} T_0^5 \quad (21)$$

Note that on the one hand

$$A^* = \begin{bmatrix} C_1 C_2 & S_1 C_2 & -S_2 & d_1 S_2 - a_2 \\ -C_1 S_2 & -S_1 S_2 & -C_2 & d_1 C_2 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (22)$$

and on the other hand

$$A^* = \begin{bmatrix} C_{34} C_5 & -C_{34} S_5 & -S_{34} & -d_5 S_{34} - a_4 C_{34} + a_3 C_3 \\ S_{34} C_5 & -S_{34} S_5 & C_{34} & +d_5 C_{34} + a_4 S_{34} + a_3 S_3 \\ -S_5 & -C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (23)$$

Thus

$$A_{11}^* = C_1 C_2 n_x + S_1 C_2 n_y - S_2 n_z = C_{34} C_5, \quad (24)$$

$$A_{12}^* = C_1 C_2 o_x + S_1 C_2 o_y - S_2 o_z = -C_{34} S_5, \quad (25)$$

$$A_{13}^* = C_1 C_2 a_x + S_1 C_2 a_y - S_2 a_z = -S_{34}, \quad (26)$$

$$A_{14}^* = C_1 C_2 p_x + S_1 C_2 p_y - S_2 p_z + d_1 S_2 - a_2 = -d_5 S_{34} + a_4 C_{34} + a_3 C_3, \quad (27)$$

$$A_{21}^* = -C_1 S_2 n_x - S_1 S_2 n_y - C_2 n_z = S_{34} C_5, \quad (28)$$

$$A_{22}^* = -C_1 S_2 o_x - S_1 S_2 o_y - C_2 o_z = -S_{34} S_5, \quad (29)$$

$$A_{23}^* = -C_1 S_2 a_x - S_1 S_2 a_y - C_2 a_z = C_{34}, \quad (30)$$

$$A_{24}^* = -C_1 S_2 p_x - S_1 S_2 p_y - C_2 p_z + d_1 C_2 = -d_5 C_{34} + a_4 S_{34} + a_3 S_3, \quad (31)$$

$$A_{31}^* = -S_1 n_x + C_1 n_y = -S_5, \quad (32)$$

$$A_{32}^* = -S_1 o_x + C_1 o_y = -C_5, \quad (33)$$

$$A_{33}^* = -S_1 a_x + C_1 a_y = 0, \quad (34)$$

$$A_{34}^* = -S_1 p_x + C_1 p_y = 0, \quad (35)$$

From equation (35) it is clear that

$$\theta_1 = \tan^{-1}(p_y / p_x) \quad (36)$$

$$\theta_1 = \theta_1 + 180^\circ$$

From equations (32) and (33) one finds that

$$\theta_5 = \tan^{-1} \left[\frac{p_x n_y - n_x p_y}{p_x o_y - o_x p_y} \right]. \quad (37)$$

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Multiplying equation (24) by C_2 and equation (28) by S_2 and subtracting the results, i.e., $A_{11}^*C_2 - A_{21}^*S_2$, yields

$$n_x C_1 + n_y S_1 = C_5 C_{234} \quad (38)$$

Similarly, multiplying equation (24) by S_2 and equation (28) by C_2 and adding the results, i.e., $A_{11}^*S_2 + A_{21}^*C_2$, yields

$$-n_z = C_5 S_{234} \quad (39)$$

Dividing equation (39) by equation (38) yields

$$\theta_{234} = \theta_2 + \theta_3 + \theta_4 = \tan^{-1} \left[\frac{-n_z}{n_x C_1 + n_y S_1} \right] \quad (40)$$

provided $C_5 \neq 0$, i.e., $\theta_5 \neq (n+1)\pi/2$. Dividing equation (25) by equation (24) yields

$$\frac{A_{12}^*}{A_{11}^*} = \frac{C_1 C_{20x} + S_1 C_{20y} - S_{20z}}{C_1 C_{2n_x} + S_1 C_{2n_y} - S_{2n_z}} = -\tan \theta_5 \quad (41)$$

Equation (41) then yields the following expression for θ_2 by dividing its numerator and the denominator by C_2 :

$$\theta_2 = -\tan^{-1} \left[\frac{C_1(o_x + n_x \tan \theta_5) + S_1(o_y + n_y \tan \theta_5)}{n_z \tan \theta_5 + o_z} \right] \quad (42)$$

$$\theta_2 = \theta_2 + 180^\circ$$

From equations (26) and (30) one finds that

$$\frac{A_{13}^*}{A_{23}^*} = -\tan(\theta_3 + \theta_4) = \frac{C_1 C_{2a_x} + S_1 C_{2a_y} - S_{2a_z}}{-C_1 S_{2a_x} - S_1 S_{2a_y} - C_{2a_z}} \quad (43)$$

which then yields the following expression for $(\theta_3 + \theta_4)$:

$$\theta_{34} = (\theta_3 + \theta_4) = \tan^{-1} \left[\frac{C_1 a_x + S_2 a_y - a_z \tan \theta_2}{C_1 a_x \tan \theta_2 + S_1 a_y \tan \theta_2 + a_z} \right] \quad (44)$$

From equations (27) and (31) one finds respectively that

$$a_3 C_3 = C_1 C_{2p_x} + S_1 C_{2p_y} - S_{2p_z} + d_1 S_2 - a_2 + d_5 S_{34} - a_4 C_{34} \quad (45)$$

$$a_3 S_3 = -C_1 S_{2p_x} - S_1 S_{2p_y} - C_{2p_z} + d_1 C_2 + d_5 C_{34} - a_4 S_{34} \quad (46)$$

Dividing equation (46) by equation (45) yields

$$\theta_3 = \tan^{-1} \left[\frac{d_1 C_2 + d_5 C_{34} - a_4 S_{34} - C_1 S_{2p_x} - S_1 S_{2p_y} - C_{2p_z}}{d_1 S_2 + d_5 S_{34} - a_4 C_{34} - a_2 + C_1 C_{2p_x} + S_1 C_{2p_y} - S_{2p_z}} \right] \quad (47)$$

Now from equations (44) and (47) one finds that

$$\theta_4 = \theta_{34} - \theta_3 \quad (48)$$

and thus the inverse kinematics solution set is completely determined.

Alternatively, one may solve the inverse kinematics problem by simply expanding equation (16). Thus, one finds the following expanded expressions

$$n_x = C_1 C_{234} C_5 + S_1 S_5 \quad (49)$$

$$o_x = -C_1 C_{234} S_5 + S_1 C_5 \quad (50)$$

$$a_x = -C_1 S_{234} \quad (51)$$

$$p_x = -d_5 C_1 S_{234} + a_4 C_1 C_{234} + a_3 C_1 C_{23} + a_2 C_1 C_2 \quad (52)$$

$$n_y = S_1 C_{234} C_5 - C_1 S_5 \quad (53)$$

$$o_y = -S_1 C_{234} S_5 - C_1 C_5 \quad (54)$$

$$a_y = S_1 S_{234} \quad (55)$$

$$p_y = -d_5 S_1 S_{234} + a_4 S_1 C_{234} + a_3 S_1 C_{23} + a_2 S_1 C_2 \quad (56)$$

$$n_z = -S_{234} C_5 \quad (57)$$

$$o_z = S_{234} S_5 \quad (58)$$

$$a_z = -C_{234} \quad (59)$$

$$p_z = -d_5 C_{234} - a_4 S_{234} - a_3 S_{23} - a_2 S_2 + d_1 \quad (60)$$

From equation (59)

$$\theta_2 + \theta_3 + \theta_4 = \cos^{-1}(-a_z) \quad (61)$$

Multiplying equation (52) by S_1 and equation (56) by C_1 and subtracting the results, i.e., $S_1 p_x - C_1 p_y$, yields

$$S_1 p_x - C_1 p_y = 0 \quad \text{or} \quad \theta_1 = \tan^{-1}(p_y/p_x) \quad (62)$$

Now if $S_{234} \neq 0$ then from equations (57) and (58) one finds that

$$\theta_5 = \tan^{-1}(-o_z/n_z), \quad S_{234} \neq 0 \quad (63)$$

But if $S_{234} = 0$ (degenerate case), then $\theta_2 + \theta_3 + \theta_4 = 0, \pm\pi$ (vertical approach for Rhino). Thus, $C_{234} = 1, \pm 1$ respectively, and from equations (49) and (50)

$$n_x = \pm C_1 C_5 + S_1 S_5 = \cos(\theta_1 \mp \theta_5) \quad \cos(\theta_1 \mp \theta_5), \quad (64)$$

$$o_x = \mp C_1 S_5 + S_1 C_5 = \sin(\theta_1 \mp \theta_5) \quad \sin(\theta_1 \mp \theta_5), \quad (65)$$

and therefore

$$\theta_1 \mp \theta_5 = \tan^{-1}(o_x/n_x), \quad \text{of} \quad S_{234} = 0 \quad (66)$$

From equations (52) and (56) one may easily show that

$$C_1 p_x + S_1 p_y = -d_5 S_{234} + a_4 C_{234} + a_2 C_2 + a_3 C_{23} \quad (67)$$

Now from equations (60) and (67) it may readily be shown that

$$a_2 S_2 + a_3 S_{23} = d_1 - d_5 C_{234} - a_4 S_{234} - p_z = \alpha \quad (68)$$

$$a_2 C_2 + a_3 C_{23} = C_1 p_x + S_1 p_y + d_5 S_{234} - a_4 C_{234} = \beta \quad (69)$$

where α and β are known quantities because $\theta_2 + \theta_3 + \theta_4$ is known already.

From equations (68) and (69) one may easily show that

$$\alpha^2 + \beta^2 = a_2^2 + a_3^2 + 2a_2 a_3 C_3 \quad (70)$$

Such that

$$\theta_3 = \cos^{-1} \left[\frac{\alpha^2 + \beta^2 - a_2^2 - a_3^2}{2a_2 a_3} \right] \quad (71)$$

Here the arc-cosine solution is acceptable because it can easily be shown that the argument can not exceed ± 1 . Generally speaking, arc-tangent solutions are preferred because they cover the whole domain from $-\alpha$ to $+\alpha$ for their argument. Now,

expanding equations (68) and (69) one obtains

$$a_3(S_2C_3 + C_2S_3) + a_2S_2 = \alpha \quad (72)$$

$$a_3(C_2C_3 - S_2S_3) + a_2C_2 = \beta \quad (73)$$

Multiply equation (72) by S_2 and equation (73) by C_2 and add to obtain

$$a_3C_3 + a_2 = \alpha S_2 + \beta C_2 \quad (74)$$

Now multiply equation (72) by C_2 and equation (73) by S_2 and subtract to obtain

$$a_3S_3 = \alpha C_2 - \beta S_2 \quad (75)$$

Multiplying equation (74) by β and equation (75) by α and adding, one obtains

$$\beta(a_3C_3 + a_2) + \alpha a_3S_3 = (\alpha^2 + \beta^2)C_2 \quad (76)$$

Multiplying equation (74) by α and equation (75) by β and subtracting one obtains

$$\alpha(a_3C_3 + a_2) - \beta a_3S_3 = (\alpha^2 + \beta^2)S_2 \quad (77)$$

Now from equations (76) and (77) it is readily found that

$$\theta_2 = \tan^{-1} \left[\frac{\alpha(a_3C_3 + a_2) - \beta a_3S_3}{\beta(a_3C_3 + a_2) + \alpha a_3S_3} \right] \quad (78)$$

So that from equations (61), (71), and (78) one finds that

$$\theta_4 = \cos^{-1}(-a_z) - \cos^{-1} \left[\frac{\alpha^2 + \beta^2 - a_2^2 - a_3^2}{2a_2a_3} \right] - \tan^{-1} \left[\frac{\alpha(a_3C_3 + a_2) - \beta a_3S_3}{\beta(a_3C_3 + a_2) + \alpha a_3S_3} \right] \quad (79)$$

This completes the inverse kinematics solutions for the Rhino XR-2.

Table 2 summarizes the inverse kinematics solutions as well as their valid ranges for all motions of Rhino XR-2 robot.

TABLE 2
Inverse Kinematics
Solution for the
Rhino Robot

Joint	Solutions	Valid Range
1	$\theta_1 = \tan^{-1}(p_y/p_x)$	$-180^\circ \leq \theta_1 \leq 180^\circ$
2	$\theta_2 = \tan^{-1} \left[\frac{\alpha(a_3C_3 + a_2) - \beta a_3S_3}{\beta(a_3C_3 + a_2) + \alpha a_3S_3} \right]$	$-90^\circ \leq \theta_2 \leq 90^\circ$
3	$\theta_3 = \cos^{-1} \left[\frac{\alpha^2 + \beta^2 - a_2^2 - a_3^2}{2a_2a_3} \right]$	$0^\circ \leq \theta_3 \leq 180^\circ$
4	$\theta_4 = \cos^{-1}(-a_z) - \cos^{-1} \left[\frac{\alpha^2 + \beta^2 - a_2^2 - a_3^2}{2a_2a_3} \right] - \tan^{-1} \left[\frac{\alpha(a_3C_3 + a_2) - \beta a_3S_3}{\beta(a_3C_3 + a_2) + \alpha a_3S_3} \right]$	$-270^\circ \leq \theta_4 \leq 90^\circ$
5	$\theta_5 = \tan^{-1}(-o_z/n_z)$, if $S_{234} \neq 0$ $\pm \theta_5 = \tan^{-1}(p_y/p_x) - \tan^{-1}(o_x/n_x)$, if $S_{234} = 0$	$-180^\circ \leq \theta_5 \leq 180^\circ$

The Rhino XR-2 firmware design is such that when the shoulder motor is activated to produce a rotation $\theta_2 = \delta$, the elbow motor is simultaneously activated to produce a rotation of $\theta_3 = -\delta$. Thus, $\theta_2 + \theta_3$ remains fixed when the shoulder motor is activated. Thus, the orientation of the forearm relative to the base will be unaffected by the shoulder motor. Similarly, when the elbow motor is activated to produce a rotation of $\theta_3 = \delta$, the wrist pitch motor is simultaneously activated to produce a rotation of $\theta_4 = -\delta$. Thus, the Rhino operation is basically such that $\theta_2 + \theta_3 + \theta_4$ remains constant when either the shoulder motor or the elbow motor is activated. This means the orientation of the wrist relative to the base will be unaffected by either the shoulder motor or the elbow motor. Thus, the wrist pitch motor may be used to set the sum $\theta_2 + \theta_3 + \theta_4$, then the elbow motor can be used to set $\theta_2 + \theta_3$, and, finally, the shoulder motor may be used to set θ_2 . Table 3 below displays the angular control ranges of the Rhino XR-2 motors.

TABLE 3
Angular Control of Robot Motors

Motor	Function	Angular Control
A	Fingers	--
B	Wrist Roll	θ_5
C	Wrist Pitch	$\theta_2 + \theta_3 + \theta_4$
D	Elbow	$\theta_2 + \theta_3$
E	Shoulder	θ_2
F	Body (Base)	θ_1
G	Carrousel, etc.	--
H	Spare, etc.	--

Now consider the home joint vector θ^h , i.e., $\theta^h = (0^\circ, -90^\circ, 90^\circ, -90^\circ, 0^\circ)^T$. Let motor angles be denoted by $\phi^m \equiv (\phi_1, \phi_2, \phi_3, \phi_4, \phi_5)^T$. Note that $\phi_1 = \theta_1$, $\phi_2 = \theta_2$, $\phi_3 = \theta_3 + (\theta_2 - \theta_2^h) = \theta_2 + \theta_3 + 90^\circ$, $\phi_4 = \theta_4 + (\theta_2 - \theta_2^h) + (\theta_3 - \theta_3^h)$

$= \theta_2 + \theta_3 + \theta_4 = \theta_{234}$ and $\phi_5 = \theta_5$. It is clear that

$$\begin{bmatrix} 0 \\ -90 \\ 90 \\ -90 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -90 \\ 90 \\ -90 \\ 0 \end{bmatrix} = \theta^h \quad (80)$$

Furthermore,

$$\Delta\phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \Delta\theta + \begin{bmatrix} 0 \\ 0 \\ 90 \\ 0 \\ 0 \end{bmatrix}, \quad (81)$$

where $\Delta\phi$ and $\Delta\theta$ are incremental changes in joint motor angles and the joint angles respectively.

The following algorithm can be used for the inverse problem:

Algorithm 2. 1. Calculate $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ using Table 2;

2. Drive the wrist motor B to the position $\phi_5 = \theta_5$;

3. Drive the wrist pitch motor C to the position $\phi_4 = \theta_2 + \theta_3 + \theta_4 = 0$;

4. Drive the elbow motor D to the position $\phi_3 = \theta_2 + \theta_3$;

5. Drive the shoulder motor E to the position $\phi_2 = \theta_2$; and

6. Drive the base motor F to the position $\phi_1 = \theta_1$.

The above procedure can be employed to calculate the desired solutions for the joint vector θ and to translate that in terms of the motor angles. The motor angles can then be activated by appropriate joint commands through the controller. A flow chart can be constructed for these purposes (Figure 3).

Based on the algorithm given in Figure 3, a computer program can be established to actually implement the inverse kinematics solutions for the motion of the Rhino XR-2 robot. This program is available on a floppy disk for any suitable microcomputer and can be obtained from the author on request.

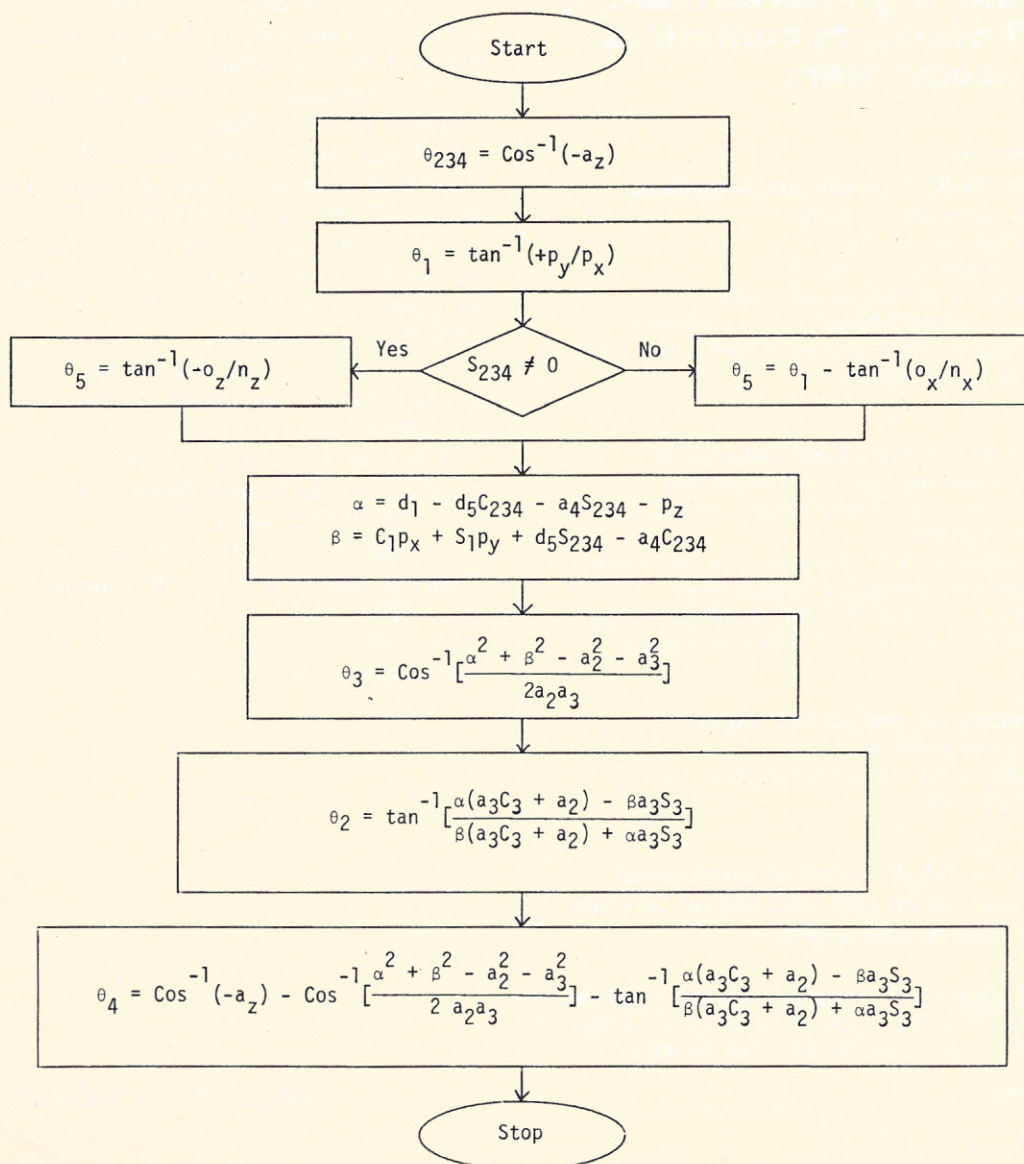
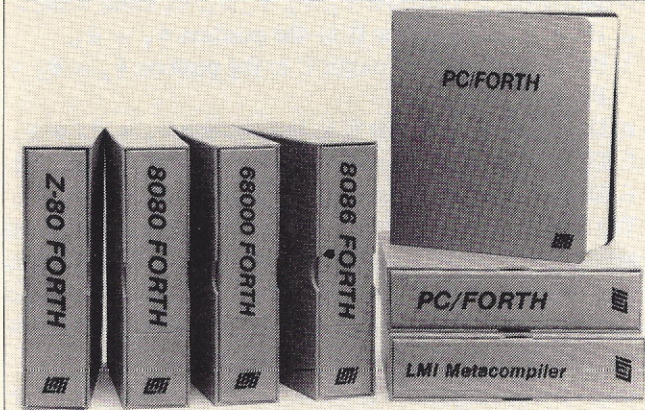


Figure 3. Based on this algorithm, a computer program can be established to implement the inverse kinematics solutions for the Rhino XR-2 robot.

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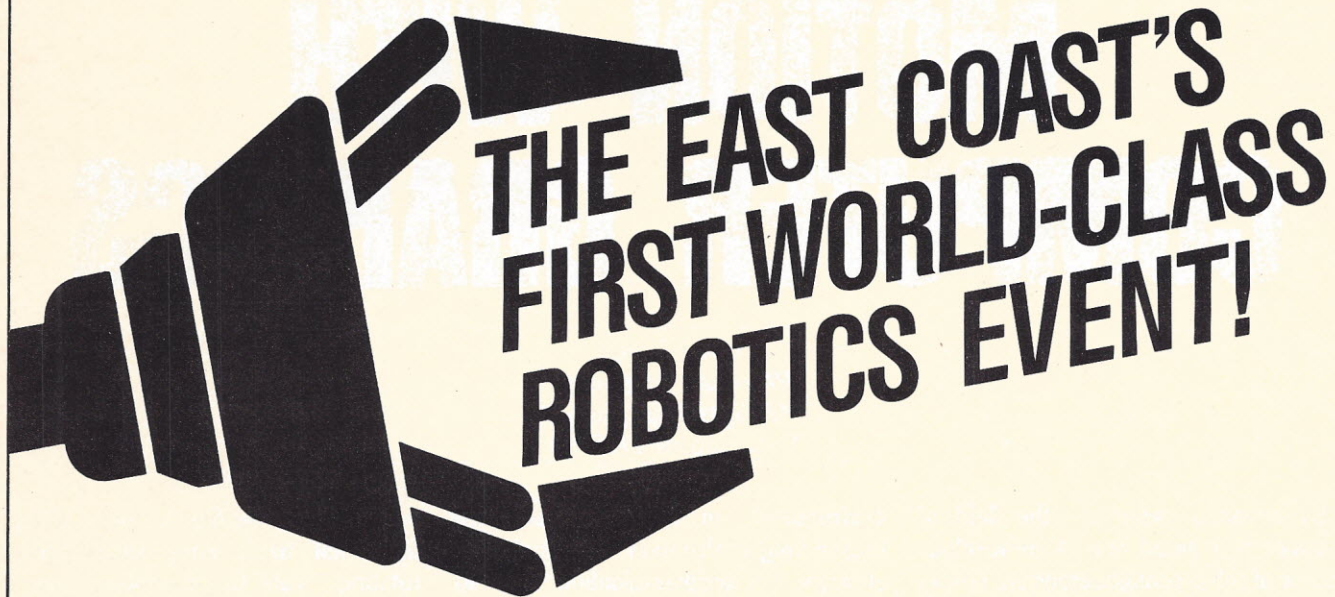
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PREVIEWING ROBOTIC MOTION WITH COMPUTER GRAPHICS

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By anyone's definition, the field of robotics is a broad one. A tremendous variety of robot configurations are now in use or being designed. They range from simple, numerically controlled positioning devices to fully autonomous, mobile data-gathering devices sophisticated enough to collect information about their surrounding environments and act on that information. Between these extremes there exists a large array of jointed arms devised for various industrial applications. In every case it is possible to use three-dimensional computer graphics as an aid in the design, programming, and guidance of these devices. Designers can use graphic systems to visualize configurations before a prototype is made. In the area of reprogramming, computer graphics can save time and even eliminate the need for access to the target device for the reprogramming stage. When designing and controlling the most complex, autonomous robots, computer graphics can be used to evaluate how the robot will react to and perform within a three-dimensional environment.

Over the last year, Wavefront Technologies has developed a program called PreView that provides a great many of the capabilities mentioned above. Although it is intended to be a general-purpose visual simulator, it incorporates some features that are particularly useful in robotics. Our design philosophy has been to provide a dynamic and interactive

environment on a high-performance engineering workstation. The goal is to shorten the design/evaluation loop so more configurations can be explored in a given time period than has been possible with previous approaches. Although computer graphics have been used in the past for various simulations, most of the software packages available have been relatively special purpose and therefore somewhat limited in the kinds of scenarios that can be simulated. Depending on the speed of the graphics hardware, PreView can render scenes of 10,000 vectors in real time (30 fps). The software is written in C and currently runs under the Unix operating system.

The basic primitives incorporated in our software are objects, viewpoints, and light sources. Objects are the "actors" in the computer-generated simulation. Associated with each object is a list of three-dimensional coordinates that describes the overall shape and size. These coordinates can be entered in a variety of ASCII and binary formats, which makes it easier to take advantage of any existing modeling software. A given simulation might have many of these objects moving independently or connected to one another. Each object may be free to move in one or more ways. These freedoms of motion can be reduced to combinations of translation, rotation, and scale in the X, Y, and Z axes. The base of a crane, for

example, may be free to move along a horizontal track while simultaneously rotating. Scale transformations can be used to describe objects such as springs or cushions that stretch in one or more dimensions. Within PreView, each separate freedom of motion is referred to as a channel. A channel can be thought of as a stream of data describing a particular motion in a particular axis such as X translation or Z rotation. Each of these channels can be used to represent a motor or an actuator in the real world. Although each motor may have its own gearing ratio or specific operational parameters, we chose to describe the motion in terms of pure transformations in three-dimensional space because this isolates the graphic previewing from the specifics of the motor attachment. If, for example, a stream of numbers must be generated in terms of pulses for stepper motors, the conversion is done as a final output step. This is directly analogous to the metafile concept used by various CAD systems where a drawing or part is represented in a generic form and then translated at output time to a format specific to a particular plotter or display device.

In order to actually display a motion sequence, there has to be a predefined vantage point from which this motion is to be seen. For maximum flexibility, PreView supports multiple viewpoints. One use of this capability is for generating top, side,

and front views of a given assembly. Viewpoints can also be attached to objects. This enables PreView to create simultaneous views of the same scene from the vantage point of the robot and a stationary or moving observer. Viewpoints can be defined as either orthographic or perspective. Perspective views will render a scene in the same way as a camera or eye would see it, that is, with progressive foreshortening with distance. When using perspective views, PreView can exactly simulate an optical system once focal length, aperture, and aspect ratio are specified. In contrast,

orthographic projections are undistorted. They allow measurements to be made directly from the projections.

The designation of light sources and their position becomes increasingly important when renditions of higher realism are required. The locations of light sources may also come into play where the amount of lighting on a given task is critical or in question. As is the case with objects and viewpoints, light sources can have motion channels associated with them. This allows us to preview a change in position or "aim" a light source.

As alluded to earlier, the objects in a scene may be connected to one another. Our method for describing this is to assign a given object to a "parent" object. Doing so tells the computer that all motion of the "offspring" object is relative to its parent. Many objects may share a common parent. In the case of a robot hand, for instance, all the fingers would have the palm of the hand as their parent. If we diagram this attachment scheme we see a hierarchical tree (Figure 1) that describes the entire connectivity of the assembly. PreView allows the user to interactively edit this hierarchy prior to display.

As is usually the case in three-dimensional computer graphics, PreView uses 4 by 4 matrices internally to describe object transformations. A single 4 by 4 matrix can represent all of the translation, rotation, and scale transformations being applied to a single object. PreView requires the user to specify the order in which the channels are combined to form the resulting matrices. This is necessary because many transformations are non-commutative; different ordering of the same transformations will yield a different visual result. In the case of a gimbal mount, for instance, the ordering of the rotations must be specified to yield a correct simulation.

In order to achieve maximum playback speed, PreView precompiles the display list into an extremely fast form that executes on a frame-by-frame basis. To quickly display a hierarchical attachment system, PreView uses an internal matrix stack. Transformations for a given object are concatenated in a single matrix, and this matrix is "pushed" onto the stack. For displaying all objects attached to that object, the matrix stack is pushed, leaving the current matrix on the top. The transformations for the sibling objects are then concatenated onto this matrix. When the next sibling object is displayed, the matrix stack is "popped" to restore the matrix that was in place for the parent object.

We have said that a scene can be thought of as a collection of objects, each controlled by one or more channels of motion. The data required to uniquely describe a specific state consists of a value for each channel (a one-dimensional vector). In order to represent motion we simply need to assemble a number of these vectors into a matrix where columns repre-

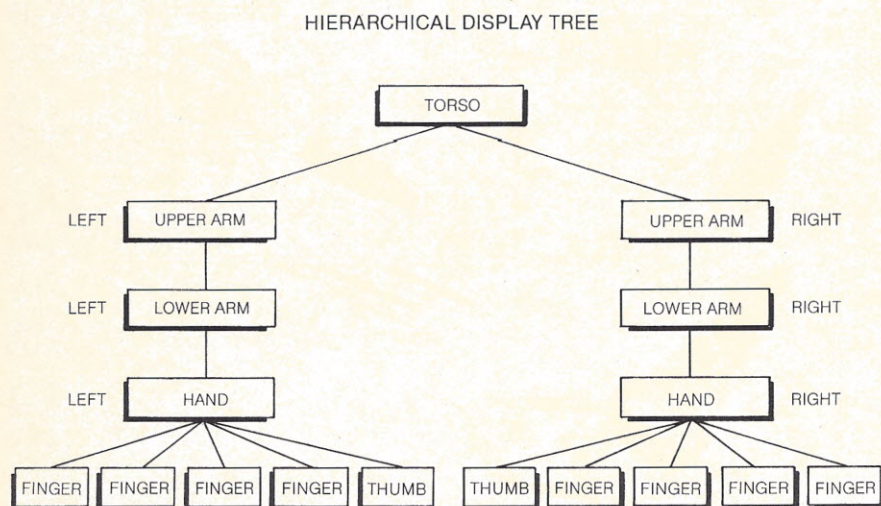


Figure 1. Objects in a given scene that are related to one another, such as the fingers of a robot hand, can be described as having a common "parent," the palm of the hand. The attachment scheme is here depicted as a hierarchical tree.

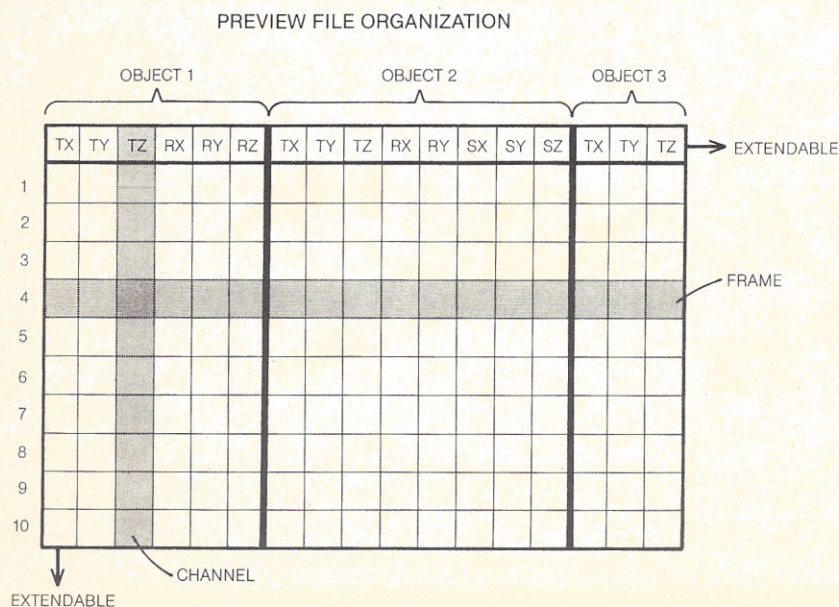


Figure 2. The PreView database is created by assembling a number of one-dimensional vectors into a matrix, in which columns represent channels of motion and rows represent frames or sample points in time.

sent channels and rows represent frames or sample points in time (Figure 2). This is roughly how a PreView database is structured. The current implementation of the program uses single-precision floating-point numbers to store channel data. To generate a dynamic display of the scene, the software indexes from frame to frame in the database, updating the matrix transformations and applying them to the associated objects for display. PreView imposes no limits on the dimensions of this matrix in terms of channels or frames. Furthermore, the database can be extended at any time to accommodate more objects or frames. The PreView database can also accommodate nongraphic channels of any type. It may be necessary, for example, to include temperature readings with several objects at specific times. Even though there may be no way to incorporate this data into the display, the readings can be synchronized and/or listed with the motion data.

In the same way that a spreadsheet program allows the accountant to modify several numbers in the matrix and observe the consequences, PreView allows the robotics designer to modify the data contained in channels and observe the visual results of that modification. Typically the user does not enter data a frame at a time but rather specifies key frames and then employs a variety of numerical techniques for making smooth transitions between those key points. At present, PreView can generate cubic splines and exponential curves as well as simple linear transitions. Each channel's key frames can be positioned at different points in time. Editing functions allow the user to move channel data from one channel to another or shift it forward or backward in time. Other commands provide for arithmetic operations (+, -, /, *) on and between channels.

PreView also allows the user to enter channel data from an external source. For instance, a set of trajectories computed by an analysis program may need to be entered into the database so the graphic results can be seen. Conversely, PreView allows for direct output of all channel data. These would be the actual data streams required to recreate the motion of an actual robot. As previously mentioned, because each channel may be representing a motor with unique properties, the PreView database stores a unique scaling factor for each channel output. In addition

to conversion factors each channel may be uniquely constrained so that only data in a specified range can be entered. The user can thus specify ranges of motion.

Figures 3 and 4 provide examples of typical PreView displays. Figure 3 shows two orthographic views and one perspective view of the same robot arm. The position and aspect ratio of each view can be specified exactly. Figure 4 shows the interactive menu for selecting key frames. Menu boxes on the right are used to

specify the current position for up to 18 channels selected from the database. Menu boxes on the left are used to specify various display modes and execute special functions. Interactive selection of these menu boxes is accomplished with a mouse or a joystick. The scale immediately below the frame is used for moving forward and backward in time. Based on the part of the scale that is touched, the rate of advance through the database can be varied from one to ten frames at a time.

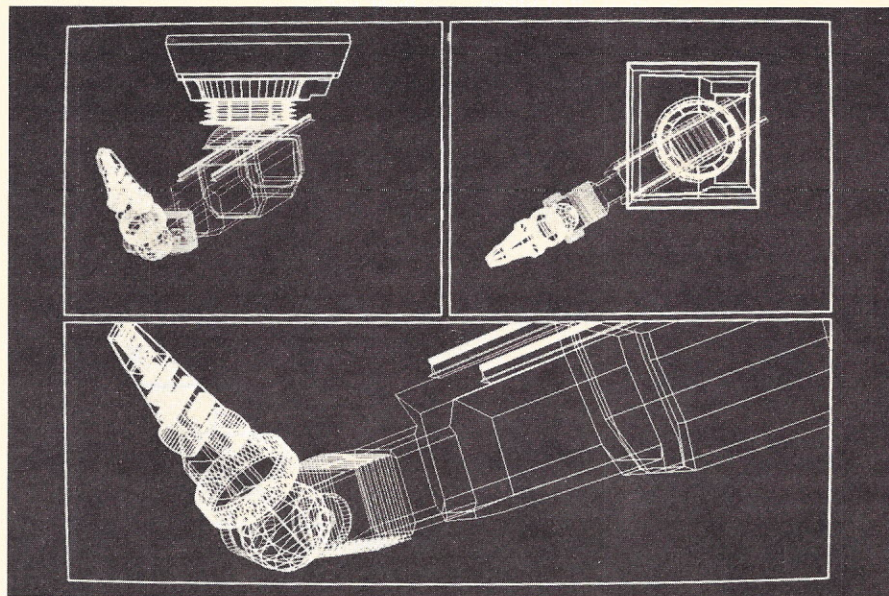


Figure 3. The same robot arm is shown in two orthographic and one perspective view.

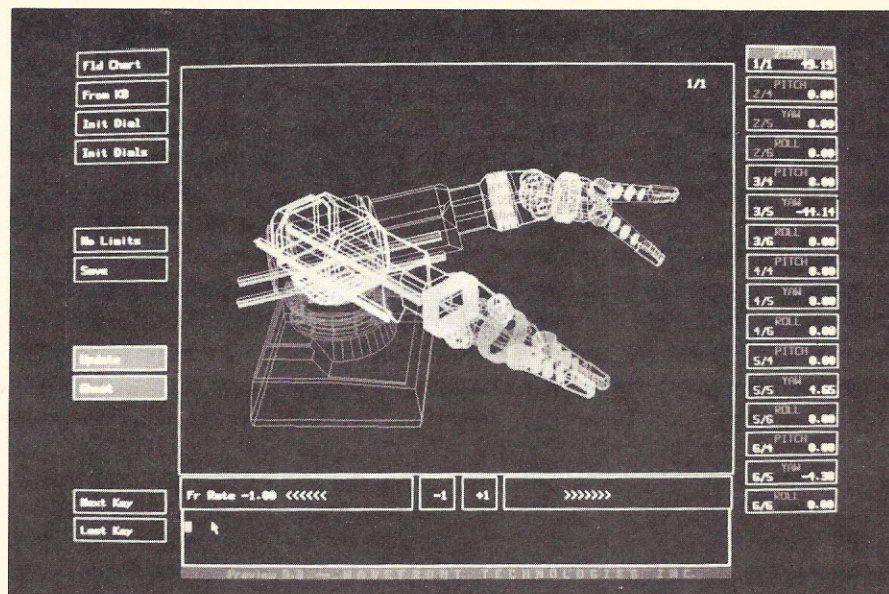


Figure 4. An interactive menu is used for selecting key frames. The menu boxes on the left specify various display modes and execute special functions and those on the right specify the current position for up to 18 channels selected from the database. The scale below the frame is used for moving forward and backward in time.

Frame #	YTRAN in. 1/ 2	ZTRAN in. 1/ 3	PITCH deg. 1/ 4	YAW deg. 1/ 5	ROLL deg. 1/ 6	XTRAN in. 2/ 1	YTRAN in. 2/ 2	ZTRAN in. 2/ 3
1	2.2366	8.8306	0.0000	43.945	89.915	0.0276	0.0000	0.0000
2	2.2117	9.0294	0.0000	43.945	89.915	0.0276	0.0000	0.0000
3	2.1872	9.2256	0.0000	43.945	89.915	0.0276	0.0000	0.0000
4	2.1629	9.4191	0.0000	43.945	89.915	0.0276	0.0000	0.0000
5	2.1390	9.6102	0.0000	43.945	89.915	0.0276	0.0000	0.0000
6	2.1154	9.7986	0.0000	43.945	89.915	0.0276	0.0000	0.0000
7	2.0922	9.9846	0.0000	43.945	89.915	0.0276	0.0000	0.0000
8	2.0692	10.168	0.0000	43.945	89.915	0.0276	0.0000	0.0000
9	2.0466	10.348	0.0000	43.945	89.915	0.0276	0.0000	0.0000
10	2.0242	10.527	0.0000	43.945	89.915	0.0276	0.0000	0.0000
11	2.0022	10.703	0.0000	43.945	89.915	0.0276	0.0000	0.0000
12	1.9805	10.876	0.0000	43.945	89.915	0.0276	0.0000	0.0000
13	1.9591	11.047	0.0000	43.945	89.915	0.0276	0.0000	0.0000
14	1.9381	11.215	0.0000	43.945	89.915	0.0276	0.0000	0.0000
15	1.9173	11.381	0.0000	43.945	89.915	0.0276	0.0000	0.0000

Figure 5. PreView's numeric output takes the form of a computer printout. Several channels of data appear in this example.

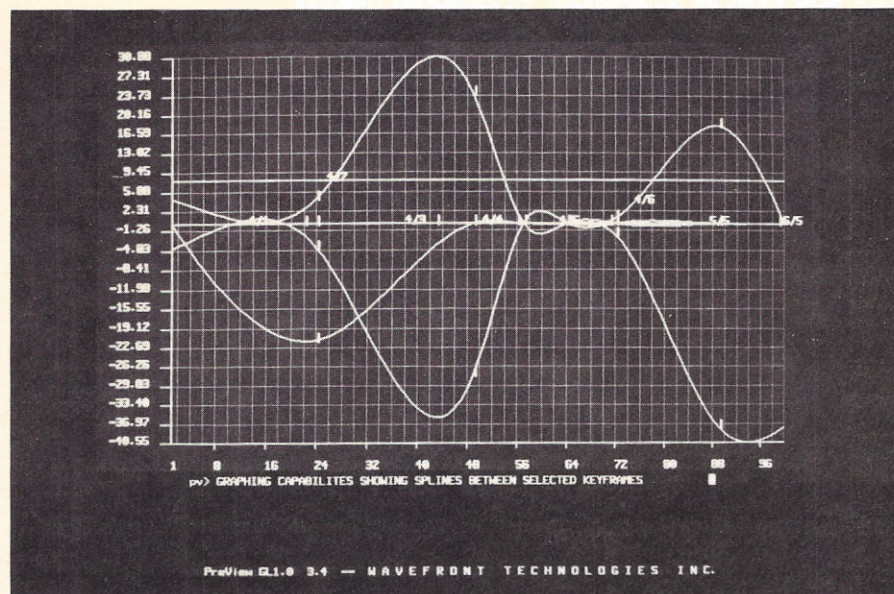


Figure 6. Channel data versus time can be represented by two-dimensional graphs. Channel data appears along the left-hand scale and time or frames appear along the bottom. Tick marks indicate key frames.

Figure 5 shows a numeric listing of several channels of data.

In addition to three-dimensional displays and alphanumeric listings, PreView can create two-dimensional graphs of channel data versus time. Figure 6 shows such a graph with channel data along the left-hand scale and time or frames across the bottom. Key frames are shown with tick marks on the various curves.

PreView has been successful in helping to visualize a wide variety of design problems, and work is continuing on additional features. Various types of special-purpose routines will be required to compute valid orientations in cases where objects are multiply constrained, such as a piston that is constrained by both the cylinder and the

crankshaft. Consideration has been given to the input of real-time data into the database. Increased realism in graphic images will become increasingly important as engineers begin to rely more heavily on digital computer graphics in presentation as well as in the design phases of a project.

Bill Kovacs is vice president of research and development at Wavefront Technologies.

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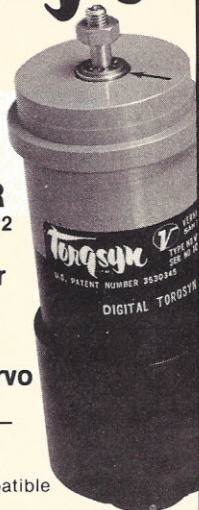
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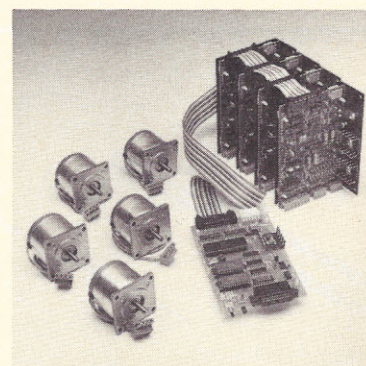
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INVERSE ROBOT KINEMATICS DERIVED FROM PLANES OF MOVEMENT

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The three joints of most use in a robot are shown in Figure 1 together with their symbolic equivalents. Suppose a link with a sliding joint is connected from point u and is directed along vector $\alpha = [\alpha_1 \alpha_2 \alpha_3]$, then the end of the link is at point $v = u + \sigma\alpha$, where $1 \leq \sigma \leq \bar{\sigma}$ and $\bar{\sigma}$ is usually less than 2, and $\{\alpha_1^2 + \alpha_2^2 + \alpha_3^2\}^{1/2}$ is the shortest length of the link. Strictly speaking, α and v are not conformable since vector α is a three tuple and v is a four tuple, but if α is considered to be a four tuple with zero fourth entry, the inconsistency is avoided (or $v - u$ treated as a three tuple since its fourth entry is zero). This fiction is used for compactness.

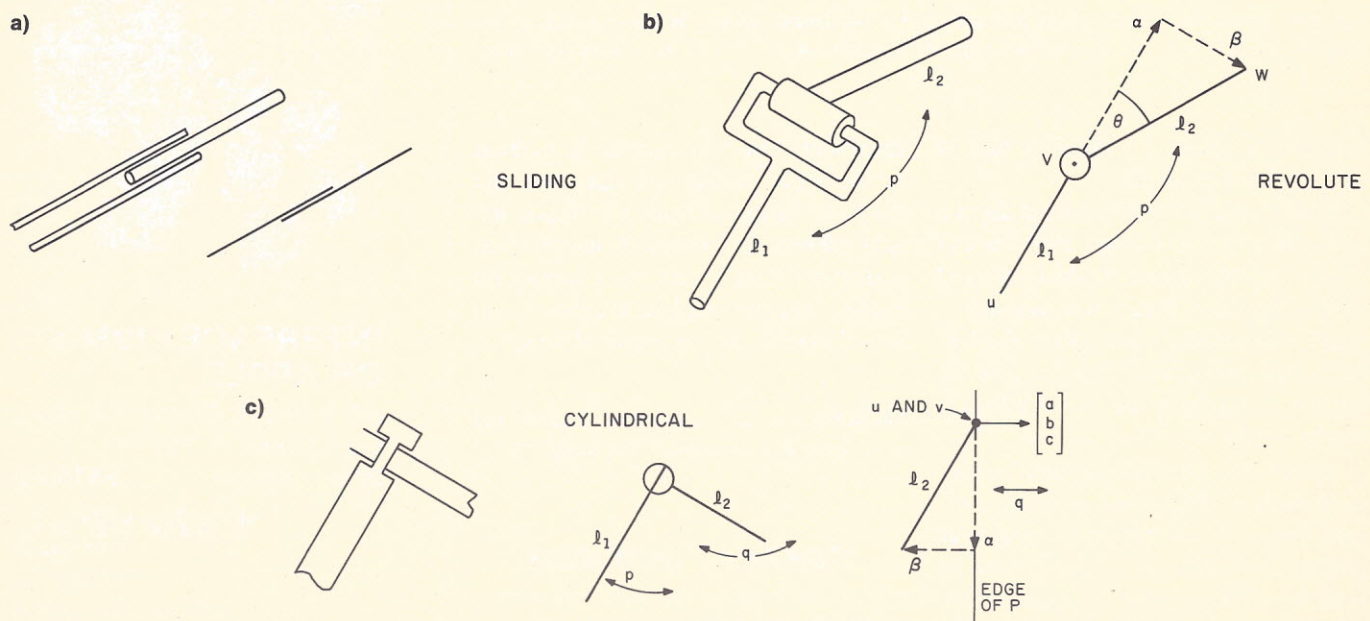


Figure 1. The sliding, revolute, and cylindrical forms of robot joints are shown, along with their geometric equivalents.

Suppose a joint involving rotation connects links ℓ_1 and ℓ_2 , and the plane of movement p of the first link ℓ_1 is known, then link ℓ_2 moves in plane q where

- (i) $p = q$ if the joint is revolute,
- (ii) $p \cdot q = 0$ if the joint is cylindrical.

The revolute joint is shown as a circle joining the two links with a dot in the middle to indicate the direction of the hinge pin. The cylindrical joint is shown as a circle with the pin of the hinge joint in line with one of the links; the other link will always be orthogonal to the first link. Notice that the cylindrical joint has orientation and can be connected in one of two ways. We consider the joint and the link to which it is connected as composing a degree of freedom. Other joints used in robotics can always be expressed in terms of the three joints defined above. There are other ways of designating robotic joints [1,2], but the notation adopted is consistent with the philosophy of the planes of movement.

THE REVOLUTE JOINT

We define the movement of a link following a joint of a robot in terms of the plane of movement of the link preceding the joint. Consider the revolute joint of Figure 1b. We know the location of the end points u and v of link ℓ_1 which moves in plane p and wish to determine the location of the end w of link ℓ_2 in terms of link ℓ_1 and angle of rotation θ . From Figure 1b we see $|\alpha| = \ell_2 \cos \theta$ and $|\beta| = \ell_2 \sin \theta$. α is in the direction of link ℓ_1 and β , α , and the vector partially specifying plane p are mutually orthogonal, so

$$\alpha = (v-u) \cos \theta \text{ and } \beta = k \begin{bmatrix} a \\ b \\ c \end{bmatrix} \times \alpha \text{ where } k \text{ is a scalar and vector } \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ is directed into the paper in Figure 1b, and } k =$$

$\tan \theta$. The order of the cross-product ensures that a clockwise rotation is positive.

Thus

$$\begin{aligned} w-v &= \alpha + \beta = \alpha + \tan \theta \begin{bmatrix} a \\ b \\ c \end{bmatrix} \times \alpha = \frac{\ell_2}{\ell_1} \left\{ \cos \theta + \sin \theta \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix} \right\} (v-u) \\ &= \frac{\ell_2}{\ell_1} \begin{bmatrix} \cos \theta & -c \sin \theta & b \sin \theta \\ c \sin \theta & \cos \theta & -a \sin \theta \\ -b \sin \theta & a \sin \theta & \cos \theta \end{bmatrix} (v-u) = R(v-u) \end{aligned} \quad (1)$$

If $R(\theta) = R$, then $R(-\theta)R(\theta)v = v$ for any v .

THE CYLINDRICAL JOINT

The cylindrical joint is connected as shown in Figure 1c. Link ℓ_1 moves in plane $p = [a \ b \ c \ d]$ and link ℓ_2 moves in plane $q = [e \ f \ g \ h]$, where p and q are orthogonal. The projection of link ℓ_2 on p is vector α , and for $\psi = 0$, w is on p and $w-v = \alpha$. q is defined in part by the direction of link ℓ_1 , i.e.

$$\begin{bmatrix} e \\ f \\ g \end{bmatrix} = \frac{1}{\ell_1} (v-u)$$

$$\text{From the geometry, } \beta = -\ell_2 \sin \psi \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\alpha = \ell_2 \cos \psi \begin{bmatrix} e \\ f \\ g \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = -\frac{\ell_2}{\ell_1} \cos \psi \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} (v-u), \quad (2)$$

and $w = v + \alpha + \beta$.

The preceding analysis assumes we know p , u , v , and ψ , and we seek w and q . If we want to use the cylindrical joint the other way around (given q , w , v , and ψ and so seek to determine p and u), we rewrite the equations as

$$w-v = \ell_2 \left\{ \cos \psi \begin{bmatrix} 0 & -g & f \\ g & 0 & -e \\ -f & e & 0 \end{bmatrix} - \sin \psi \cdot I \right\} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\text{so } \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{-1}{\ell_2} \begin{bmatrix} \sin \psi & g \cos \psi & -f \cos \psi \\ -g \cos \psi & \sin \psi & e \cos \psi \\ f \cos \psi & -e \cos \psi & \sin \psi \end{bmatrix}^{-1} (w-v) \quad (3)$$

$$= \frac{-1}{\ell_2} \begin{bmatrix} \cos \phi & -g \sin \phi & f \sin \phi \\ g \sin \phi & \cos \phi & -e \sin \phi \\ -f \sin \phi & e \sin \phi & \cos \phi \end{bmatrix} (w-v)$$

where $\phi = 90 - \psi$. Point $u = v - \ell_1 \begin{bmatrix} e \\ f \\ g \end{bmatrix}$

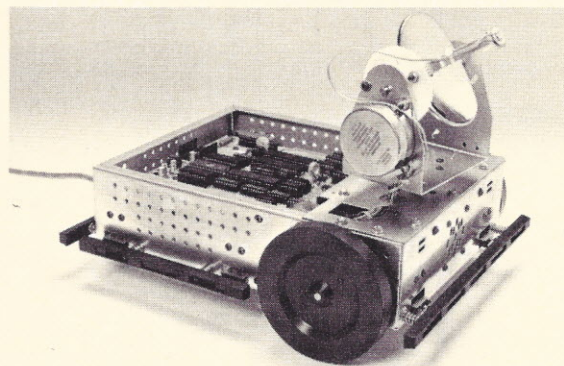
and plane p is completed with $d = -(au_x + bu_y + cu_z)$.

ROBOTIC MOVEMENT IN PLANES

To avoid any ambiguity, the following angular notation is adopted: θ refers to a revolute joint, ϕ refers to a cylindrical joint connected so that the down-link (closer to the base) moves in the same plane as the up-link (closer to the end effector), ψ refers to a cylindrical joint connected so that the down-link moves in a plane orthogonal to the up-link, and σ refers to a sliding link.

Figure 2 shows the joint combinations that produce a few of the many 3 degrees of freedom robots and the planes of movement of the links. We can make some general observations about robots. It is necessary but not sufficient for the robot to move in two independent planes in order for the end effector to reach any point in three-dimensional space. If in plane p_i a joint or end effector covers a finite area (assume that at one end of the chain of links that move in p_i the prior link that does not move in p_i is fixed, then the joint or end effector at the other end of the chain of links must move over some finite area), then we say that p_i has area and designate it as $\bar{p}_i$. This occurs whenever a chain of two or more joints and links are in the same plane.

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To reach any point in three-dimensional space, it is necessary that the robot links move in (a) at least two independent planes and that one of the planes has area, or (b) at least three independent planes if none of the planes has area.

These requirements mean that a minimum of 3 degrees of freedom are required to reach any point in three-dimensional space. The robots appearing second and third in Figure 2 do not have planes with area. We classify these robots as stiff—sometimes requiring large changes in the status of two joints for a moderate change in end effector position. A two-link addition as shown in Figure 3 to the 3 degrees of freedom robots of Figure 2 enables the end link to assume any position and orientation.

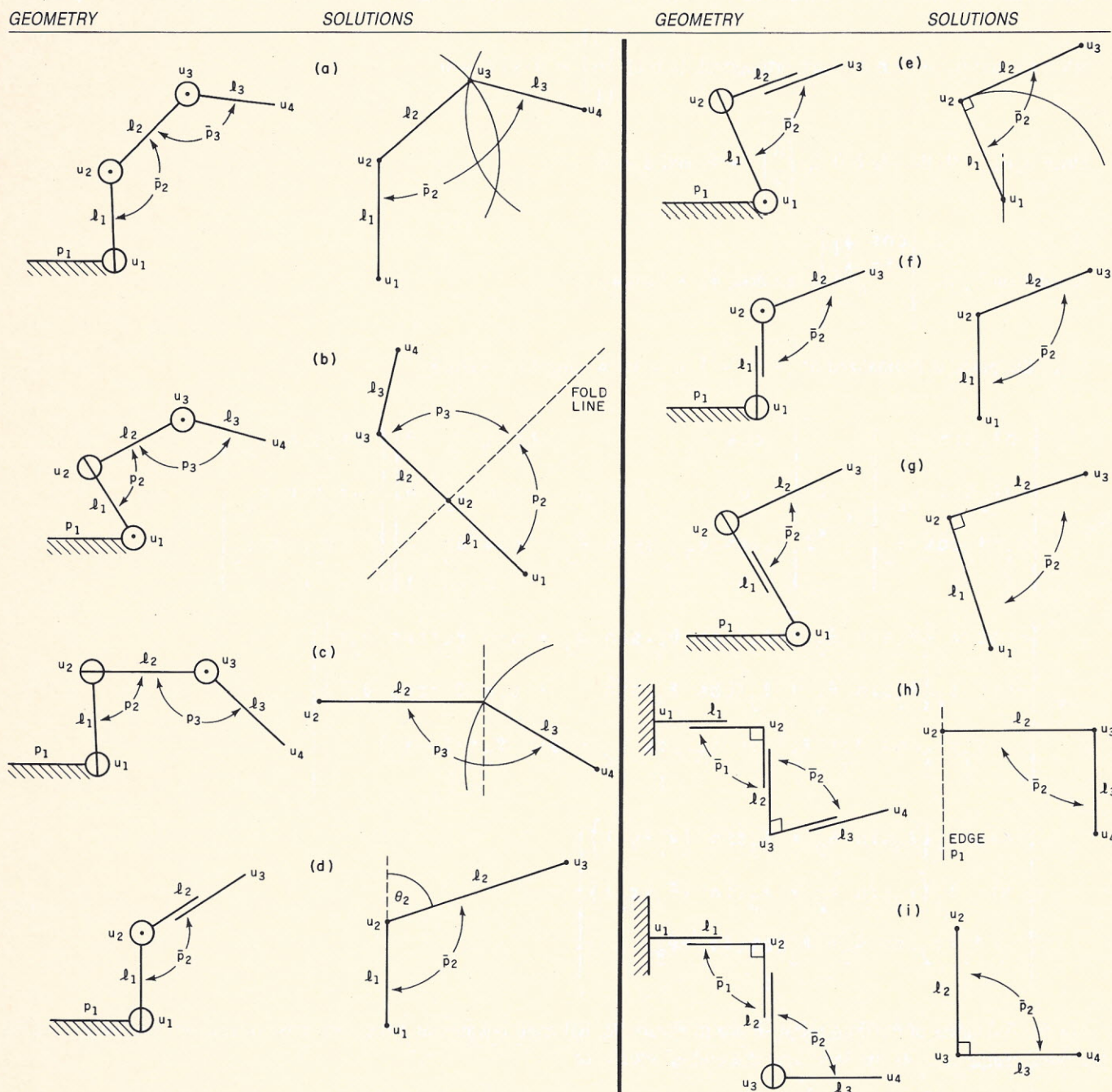


Figure 2. The joint combinations that produce many of the 3 degrees of freedom robots are depicted in the left-hand column (a-i) and their solutions appear in the right-hand column (a-i).

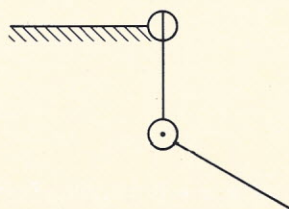


Figure 3. A two-link addition to the 3 degrees of freedom robots in Figure 2 enables the end link to assume any position or orientation.

Suppose the 3 degrees of freedom robot shown in Figure 2a, left-hand column, has its base at the origin, so $u_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

and the base orientation the x-y plane so $p_1 = [0 \ 0 \ 1 \ 0]$. We assume that for $\phi_1 = 0$ the robot moves in the x-z plane.

Since p_1 and $\bar{p}_2 = [a \ b \ c \ d]$ are orthogonal, $[a \ b \ c] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$, so $c = 0$.

Since $\bar{p}_2 u_1 = 0$, then $[a \ b \ 0 \ d] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$, and $d = 0$.

A point on $\bar{p}_2$ is $\begin{bmatrix} \cos \phi_1 \\ \sin \phi_1 \\ 0 \\ 1 \end{bmatrix}$, so $a \cos \phi_1 + b \sin \phi_1 = 0$.

Since the plane is normalized $a^2 + b^2 = 1$, $a = \sin \phi_1$ and $b = -\cos \phi_1$.

$$\begin{aligned}
 u_4 &= \begin{bmatrix} b \ell_2 \sin \theta_2 \\ -a \ell_2 \sin \theta_2 \\ \ell_1 + \ell_2 \cos \theta_2 \\ 1 \end{bmatrix} + \frac{\ell_3}{\ell_2} \begin{bmatrix} \cos \theta_3 & 0 & b \sin \theta_3 & 0 \\ 0 & \cos \theta_3 & -a \sin \theta_3 & 0 \\ -b \sin \theta_3 & a \sin \theta_3 & \cos \theta_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b \ell_2 \sin \theta_2 \\ -a \ell_2 \sin \theta_2 \\ \ell_2 \cos \theta_2 \\ 0 \end{bmatrix} \\
 &= \begin{bmatrix} -\cos \phi_1 \{ \ell_2 \sin \theta_2 + \ell_3 (\cos \theta_3 \sin \theta_2 + \sin \theta_3 \cos \theta_2) \} \\ -\sin \phi_1 \{ \ell_2 \sin \theta_2 + \ell_3 (\cos \theta_3 \sin \theta_2 + \sin \theta_3 \cos \theta_2) \} \\ \ell_1 + \ell_2 \cos \theta_2 + \ell_3 (\cos \theta_3 \cos \theta_2 - \sin \theta_3 \sin \theta_2) \\ 1 \end{bmatrix} \\
 &= \begin{bmatrix} -\cos \phi_1 \{ \ell_2 \sin \theta_2 + \ell_3 \sin (\theta_2 + \theta_3) \} \\ -\sin \phi_1 \{ \ell_2 \sin \theta_2 + \ell_3 \sin (\theta_2 + \theta_3) \} \\ \ell_1 + \ell_2 \cos \theta_2 + \ell_3 \cos (\theta_2 + \theta_3) \\ 1 \end{bmatrix} \tag{4}
 \end{aligned}$$

For the 3 degrees of freedom robot shown in Figure 2d, left-hand column, and the same base orientation and position as in the prior example, $\bar{p}_2$ is the same and the end effector is at

$$u_3 = \begin{bmatrix} -\cos \phi_1 \ell_2 \sin \theta_2 \\ -\sin \phi_1 \ell_2 \sin \theta_2 \\ \ell_1 + \ell_2 \cos \theta_2 \\ 1 \end{bmatrix} \tag{5}$$

where ℓ_2 is the length of the sliding link.

Appearing in Figure 2a, left-hand column, is a robot that is capable of reaching any point in three-dimensional space with its last link in any orientation, using the orientation linkage shown in Figure 3. This robot has one plane with area, called $\bar{p}_2$ in

the figure. u_5 can be calculated using equation (4) with θ_3 replaced by $\theta_3 + \tan^{-1}(\ell_4/\ell_3)$ and ℓ_3 replaced by

$$\sqrt{\ell_3^2 + \ell_4^2}$$

The Microbot training robot with 5 degrees of freedom shown in Figure 4 has the first three joints as previously analyzed. u_4 is the same as calculated in equation (4), and straightforward calculations establish

$$u_6 = \begin{bmatrix} -\cos \phi_1 \left\{ \ell_2 \sin \theta_2 + \ell_3 \sin (\theta_2 + \theta_3) + \sqrt{\ell_4^2 + \ell_5^2} \cdot \sin (\theta_2 + \theta_3 + \theta_4 + \tan^{-1} \frac{\ell_5}{\ell_4}) \right\} \\ -\sin \phi_1 \left\{ \ell_2 \sin \theta_2 + \ell_3 \sin (\theta_2 + \theta_3) + \sqrt{\ell_4^2 + \ell_5^2} \cdot \sin (\theta_2 + \theta_3 + \theta_4 + \tan^{-1} \frac{\ell_5}{\ell_4}) \right\} \\ \ell_1 + \ell_2 \cos \theta_2 + \ell_3 \cos (\theta_2 + \theta_3) + \sqrt{\ell_4^2 + \ell_5^2} \cdot \sin (\theta_2 + \theta_3 + \theta_4 + \tan^{-1} \frac{\ell_5}{\ell_4}) \\ 1 \end{bmatrix} \quad (5)$$

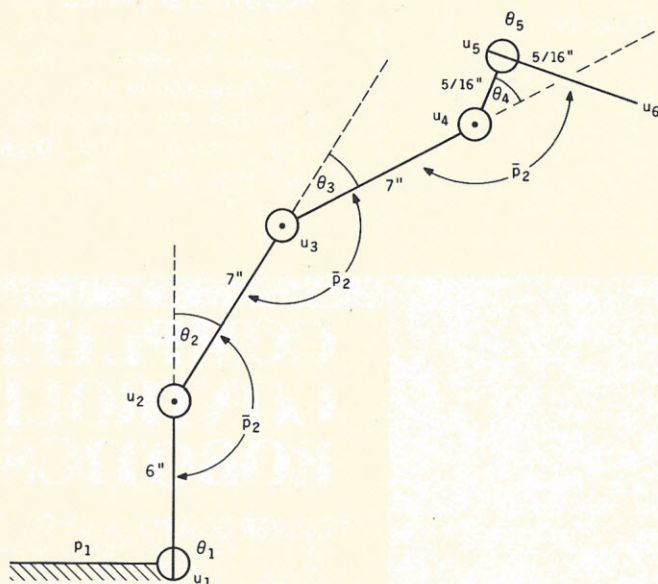


Figure 4. The Microbot training robot has 5 degrees of freedom.

For the SCARA (Selective, Compliant, Articulative Robot for Assembly) robot of Figure 5 and base orientation as given in the prior examples,

$$u_2 = \begin{bmatrix} \ell_{1b} \cos \phi_1 \\ \ell_{1b} \sin \phi_1 \\ \ell_{1a} \\ 1 \end{bmatrix}, \quad \bar{p}_2 = [0 \quad 0 \quad 1 \quad -\ell_{1a}]$$

$$u_3 = u_2 + \frac{\ell_2}{\ell_{1b}} \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left\{ u_2 - \begin{bmatrix} 0 \\ 0 \\ \ell_{1a} \\ 0 \end{bmatrix} \right\}$$

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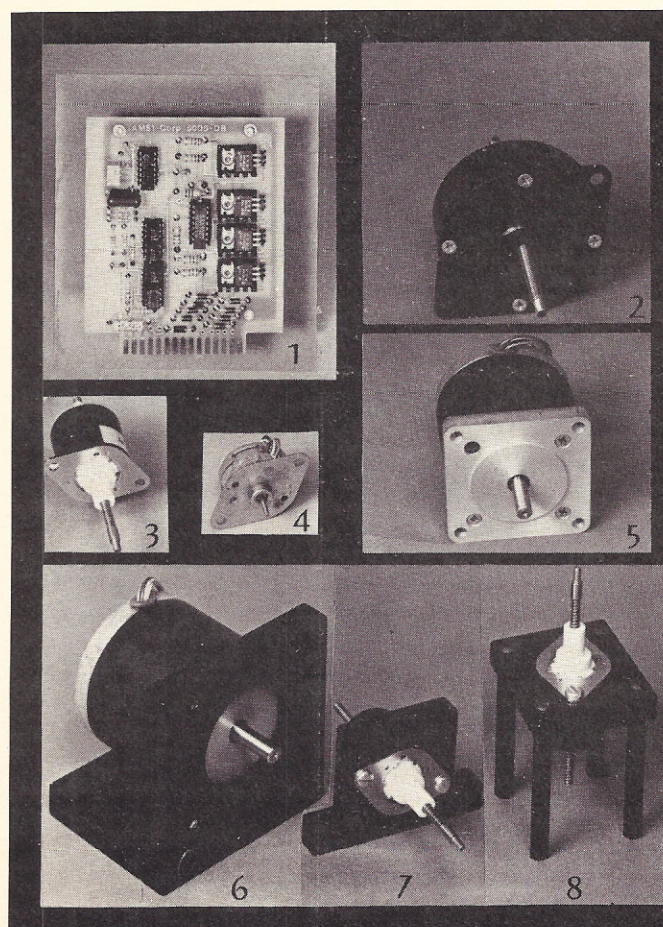
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$$\begin{aligned}
&= \begin{bmatrix} \ell_{1b} \cos \phi_1 + \ell_2 (\cos \theta_2 \cdot \cos \phi_1 - \sin \theta_2 \cdot \sin \phi_1) \\ \ell_{1b} \sin \phi_1 + \ell_2 (\cos \theta_2 \cdot \sin \phi_1 - \sin \theta_2 \cdot \cos \phi_1) \\ \ell_{1a} \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \ell_{1b} \cos \phi_1 + \ell_2 \cos (\theta_2 + \phi_1) \\ \ell_{1b} \sin \phi_1 + \ell_2 \sin (\theta_2 + \phi_1) \\ \ell_{1a} \\ 1 \end{bmatrix} \quad (7)
\end{aligned}$$

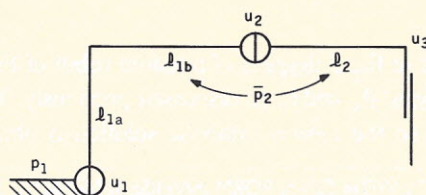


Figure 5. The symbol at u_2 indicates the direction of the pin of the SCARA robot's revolute joint.

The symbol at u_2 in Figure 5 indicates the direction of the pin of the revolute joint, so if the plane containing ℓ_{1a} is vertical, $\bar{p}_2$ is horizontal.

INVERSE KINEMATIC SOLUTIONS

The inverse kinematic problem has been given considerable attention, and for good reason [3-12]. A paint-spraying robot must maintain a fairly constant distance from the object and keep its spray nozzle approximately orthogonal to the point on the surface being sprayed. We have just defined a path control problem for the nozzle. A welding robot is required to run the weld rod along a seam, another path control problem. A SCARA robot must pick up a printed circuit board and place it in a chassis, avoiding other boards and the power supply, yet another path control problem.

For our examples of robots with 3 degrees of freedom robotic configuration geometries, the inverse kinematic solutions proceed from known p_1 , u_1 , and the end effector position. These solutions are shown geometrically in the right-hand column of Figure 2. These robots yield none, one, or two solutions. Except for the stiff robots of Figure 2, the solutions are in the planes with area. The geometric solutions indicate that closed-form algebraic solutions are possible; in other words, symbolic kinematic solutions can be obtained. Further symbolic inverse kinematic solutions can be obtained from the symbolic forward kinematic solutions.

When the robot moves in two or more independent planes, the planar representation must be interpreted correctly. The joint that causes the most problems is the cylindrical joint, since the planes of movement on either side of the joint are orthogonal. The best way to conceive of the actual movement is with fold lines. Consider the geometry shown in Figure 2b, left-hand column, where one side of the joint moves in plane $p_2 = [a \ b \ c \ d]$ and the other side moves in plane $p_3 = [e \ f \ g \ h]$, where $ae + bf + cg = 0$. The kinematic solution appears in Figure 2b, right-hand column, where the line of intersection of the two planes is shown as dotted and is called the fold line. This line is orthogonal to both links, so the links ℓ_1 and ℓ_2 are always in line in this representation. To portray the correct relative position of the planes in space, fold the paper along the fold line by 90 degrees. The movement of the cylindrical joint can be seen by considering points u_1 and u_2 to be fixed in space and the fold line rotated about the axis of link ℓ_1 .

For 4 or more degrees of freedom robots to reach any point in three-dimensional space an infinity of solutions may be possible. Such is the case with the Microbot robot. The solution that is often adopted is the one that minimizes joint movement with respect to the prior position.

The algebraic inverse kinematic solution for the 3 degrees of freedom robot of Figure 2a is found from equation (4) as follows. For the end effector at point $[x \ y \ z \ 1]^t$,

$$\ell_2 \sin \theta_2 + \ell_3 \sin (\theta_2 + \theta_3) = -y / \sin \phi_1, \text{ and}$$

$$\ell_2 \cos \theta_2 + \ell_3 \cos (\theta_2 + \theta_3) = z - \ell_1,$$

(8)

where $\phi_1 = \tan^{-1}\{y/x\}$.

Reordering and squaring both sides of equations (8) can eliminate the compound angle terms and leave

$$\frac{y}{\sin \phi_1} \sin \theta_2 - (z - \ell_1) \cos \theta_2 = \left\{ \ell_3^2 - \ell_2^2 - \frac{y^2}{\sin^2 \phi_1} - (z - \ell_1)^2 \right\} / (2 \ell_2) \quad (9)$$

where the left-hand side can be reduced to

$$\left[\frac{y^2}{\sin^2 \phi_1} + (z - \ell_1)^2 \right] \sin \left\{ \theta_2 + \tan^{-1} \frac{(z - \ell_1) \sin \phi_1}{y} \right\} \quad (10)$$

and θ_2 found explicitly.

The symbolic inverse kinematic solution of the 5 degrees of freedom robot of Figure 2a with the orientation linkage of Figure 3 is obtained from equations (8) by modifying θ_3 and ℓ_3 as discussed previously. The forward kinematic equation of the SCARA robot has a form similar to equation (4), so the inverse symbolic solution is obtained in the same way.

ROBOTS WITH OPEN-FORM INVERSE KINEMATIC SOLUTIONS

Not all robots have closed-form inverse kinematic solutions. A robot that cannot be solved in closed form (at least by the techniques employed here) is the 6 degrees of freedom robot with two planes with area and end link orientation as shown in Figure 6a. Three pieces of information are required to establish plane $\bar{p}_2 = [e \ f \ g \ h]$, and known points u_1 and u_2 provide two of these. Arbitrarily choosing the base angle ϕ_1 will complete the determination. Known points u_6 and u_7 lie on plane $\bar{p}_3 = [a \ b \ c \ d]$. The third point needed to find $\bar{p}_3$ is found by noting that $\bar{p}_2$ and $\bar{p}_3$ are orthogonal, so $[e \ f \ g][a \ b \ c]^t = 0$, which is the same as considering $[e \ f \ g \ 0]^t$ as a point on $\bar{p}_3$.

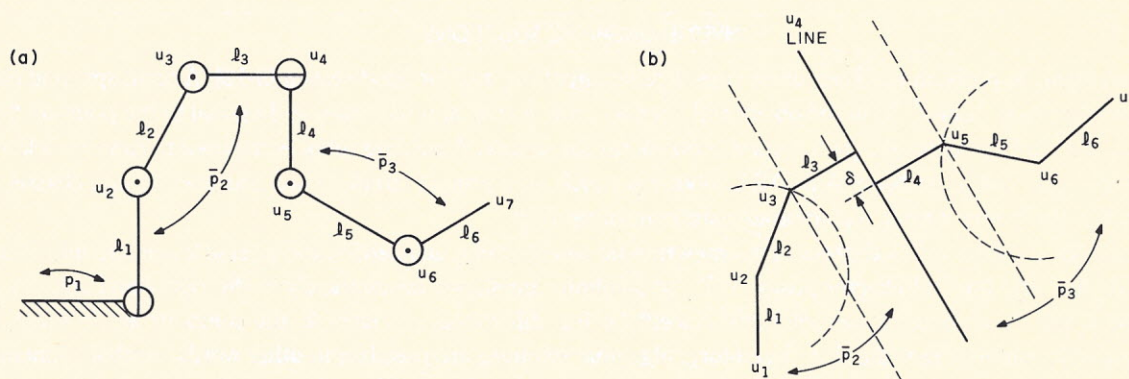


Figure 6. The 6 degrees of freedom robot shown in 6a does not have closed-form kinematic solutions. Open-form solutions for this robot are illustrated geometrically in 6b.

Point u_4 lies on both $\bar{p}_2$ and $\bar{p}_3$, and the line of intersection of these two planes will (except for a degenerate case) cross the x - y plane, the x - z plane and the y - z plane; the line of intersection is determined by one or more of the equations.

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \frac{-1}{af-be} \begin{bmatrix} f & -b \\ -e & a \end{bmatrix} \begin{bmatrix} d \\ h \end{bmatrix}, \text{ where } z_1 = 0;$$

$$\begin{bmatrix} x_2 \\ z_2 \end{bmatrix} = \frac{-1}{ag-ce} \begin{bmatrix} g & -c \\ -e & a \end{bmatrix} \begin{bmatrix} d \\ h \end{bmatrix}, \text{ where } y_2 = 0;$$

$$\begin{bmatrix} y_3 \\ z_3 \end{bmatrix} = \frac{-1}{bg-cf} \begin{bmatrix} g & -c \\ -f & b \end{bmatrix} \begin{bmatrix} d \\ h \end{bmatrix}, \text{ where } x_3 = 0.$$

The inverse kinematic solution is shown in Figure 6b, where the points in plane $\bar{p}_2$ are shown to the left of the u_4 line and the points in plane $\bar{p}_3$ are shown to the right of the u_4 line. Consider the u_4 locus to be a 90-degree fold line in the paper to get the true orientation of the planes. Notice that

$$u_3 - u_4 = \pm \begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} \text{ and } u_4 - u_5 = \pm \begin{bmatrix} e \\ f \\ g \\ 0 \end{bmatrix}$$

so links l_3 and l_4 must meet the u_4 line at 90 degrees as shown in Figure 6b. This fact establishes the straight line loci for u_3 and u_4 shown dotted in the diagram. u_3 must be on a circle of radius l_2 centered at u_2 , and u_4 must be on a circle of radius l_5 centered at u_6 . Choosing one of the two intersection points for u_3 and u_4 and drawing the normals to the u_4 line determines error $\delta = \delta(\phi_1)$. The choice of points may not be completely arbitrary since

(i) we want $\delta \rightarrow 0$, and

(ii) if the solution is part of a path control problem [10-12] where the u_6 and u_7 are a series of points, continuity dictates the choice.

A strategy must now be employed to refine ϕ_1 , to drive δ to zero. One such strategy is to change the initial value chosen for the base angle ϕ_1^1 for which δ^1 was calculated to $\phi_1^2 = \phi_1^1 + \Delta$, where Δ is small, and calculate the new value for the distance between the two points on the u_4 line given by δ^2 . Assuming δ is a linear function of ϕ_1 , we choose the base angle to be

$$\phi_1^3 = \frac{\phi_1^2 \delta^1 + \phi_1^1 \delta^2}{\delta^1 + \delta^2}$$

and continue the iteration provided $\delta^{i+1} < \delta^i$.

CONCLUSIONS

The method of specifying robotic movement developed here enables kinematic solutions to be obtained within the planes with area, greatly simplifying the calculations. In most cases, the solutions are obtained symbolically in closed form.

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GRAPHICS SIMULATION AND PROGRAMMING FOR ROBOTIC WORKCELL DESIGN

Robert R. Boren
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Shelter Rock Lane
Danbury, CT 06810

Over the years, computer aided design (CAD) technology has evolved from simple, two-dimensional drafting tools to engineering design and analysis and manufacturing control and programming from the same database. Improved software and hardware, which permit toolpath generation and N/C postprocessing to be performed from the CAD database, represent a long step toward the integration of CAD with computer aided manufacturing (CAM). A further extension of this technology is the simulation and programming of robotic workcells using interactive graphics.

Robot simulation has become an important tool for the Systems Engineering Department of Unimation Incorporated in the design and implementation of robotic systems for industry automation. Development of a robot system involves reviewing the industrial process to be addressed in the workcell, designing the best system to accomplish the task, and presenting the designs to the customer. Uses of CAD range from the design and manufacture of end-of-arm tooling to the installation of multirobot systems that interface robots, machine tools, and inspection and manufacturing equipment with flexible communication networks.

Unimation's systems engineering department employs a Computervision

CGP200X® CAD/CAM system running CADD5® 4X Rev 3/0 software. The system is used for workcell concept definition, engineering analysis, process simulation, detail-drawing generation, process planning, tool design, technical publications, and robot simulation programming. The entire process relies on CAD, from initial concept design illustrations and engineering analysis to end effector tooling and custom equipment needed for the robotic workcell.

One very important feature of the CAD system is that once a part is defined it never needs to be recreated, and it can be used for many future applications by being extracted from the parts library. Drawings stored in a CAD system can be quickly called up and revised or merged into other projects, a significant timesaver since an estimated 80 percent of new designs are based on existing documents. Because drawings in an electronic database do not deteriorate, maintenance expenses are virtually eliminated. Also, documents can be sent to remote locations via telephone lines or networks.

The system enables designers to see the results of tolerance buildup on the parts design and its effect on the entire mechanism. Fits and interferences can be precisely determined, and design changes are immediately reflected in the system

database. At any time in the design creation process, a hard copy or plot can be extracted, presenting a view of the database components from any desired orientation or scale. When a design is completed, detail drawings can be extracted for parts manufacture. The completed system can then be programmed from the same design database.

This computerized approach to engineering improves on the old trial and error manual approach in many ways. By building one centralized engineering database, the design of each component can be verified graphically and the entire workcell simulated before manufacturing begins. Designs can be changed and refined quickly, yielding a precision that eliminates the need for costly prototype construction.

In using graphical simulation to design and program robotic workcells, the following steps are necessary:

1. Create the basic workcell in a CAD database.
2. Verify that the robot can reach all required equipment.
3. Modify and refine the workcell.
4. Verify that the robot can perform the desired tasks.
5. Simulate the robot performing the desired tasks.

ROBOGRAPHICS SOFTWARE PACKAGE

The software we used for graphical simulation and programming of the robot workcell is Computervision's Robographics® system. It is composed of three modules: the robot path generator, a software kinematic model of the robot, and the robot output processor.

The robot path generator allows the designer to store necessary information about the robot and end effector and to use this information when simulating the robot in the workcell. The kinematic model is a numerical software module of the robot complete with arm solutions and the motion calculations required to drive the graphic figures and accurately represent the robot. The robot output processor, which is similar to a postprocessor, converts or translates the robot path data into the appropriate robot language.

Before the robot can be simulated and programmed on the CAD system, the designer must create a 3-D wire frame graphics fixture for each link of the robot. The designer also creates the graphic figures for the end effectors to be used on the robot and defines the appropriate control points with which the end effectors will interface with the workcell geometry. This allows the designer to build a library of robot end effectors that can be used for other applications as well.

CREATING THE WORKCELL

The typical workcell in an industrial or manufacturing environment consists of three groups of equipment: process machinery, interfacing or service equipment, and control systems. The process machinery, which performs some function on the part or material, might be a welding robot, a punch press, or an adhesive applicator. Interfacing equipment is necessary to serve and manipulate parts or material to the process machinery and may include conveyors, material handling robots, and rotary indexing table. The control equipment, which is in charge of the manufacturing process and the cell components, can be a robot controller, a welding controller, or pneumatic controls. In many cases, the robot controller is also the system controller that runs the entire workcell.

An extensive 3-D parts library of all types of equipment that might be used in a

robotic cell has been built into our CAD system. This library has grown to over 600 parts and includes various process machines, control equipment, material handling equipment, and a large assortment of robot configurations. The robotic workcell is created by taking the various pieces of equipment needed for the cell from the parts library and inserting them into the cell database. If a piece of needed equipment is not in the parts library, the CAD designer creates a 3-D model for the cell and files it in the library for future use. The equipment in the cell is then positioned according to the existing process flow.

ROBOT REACH VERIFICATION

After the initial cell layout is established in the CAD database, the next step is to verify that the robot can reach and interface with the equipment in the workcell. The "CHECK REACH" command from the Robographics software package allows the designer to quickly optimize the cell configuration by inserting the requested robot into the cell at a specified position.

The designer can then tell the robot to "reach" the necessary positions within the cell.

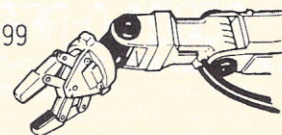
The end effector will attempt to move to the requested location and orientation in the cell and display the location coordinates and joint angles on the CRT terminal. If the robot cannot reach its destination, the appropriate joints are flagged on the CRT to indicate that they have exceeded their range of motion. At this point, while still in the CHECK REACH mode, the designer can relocate the robot, reposition the cell components, or change the orientation of the end effector so it can reach the required positions. This cycle continues until the cell is optimized for the robot selected. If it appears that the robot first selected will not reach the desired positions, a different robot can be selected from the library.

ROBOT CELL SIMULATION

When the cell layout is complete, the designer prepares it for dynamic simulation of the robot performing the necessary tasks. The command "INS ROBOT" ac-

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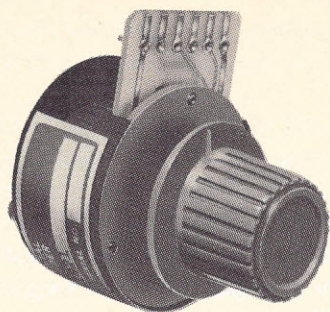
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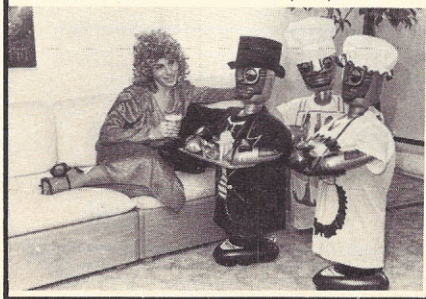
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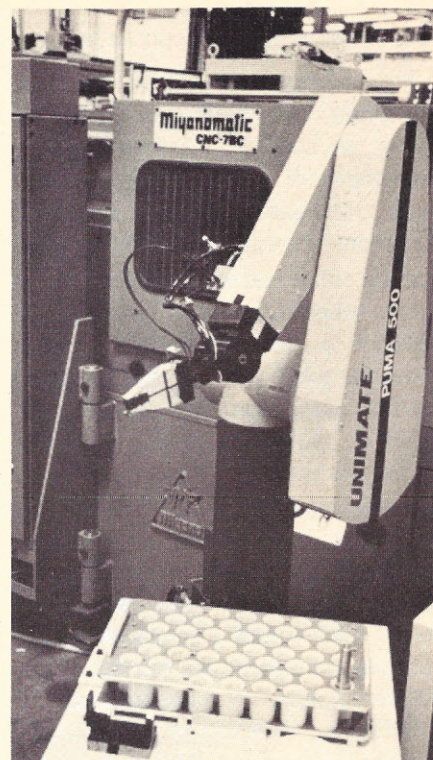
tivates robot simulation and path programming. The CAD system asks the designer for necessary parameters such as which robot and which end effector are to be used from the library selection, the orientation of the end effector and control point, and the position of the robot in the workcell. The system then graphically displays the specified robot and end effector. The joint angles for each link of the robot are also displayed on the CRT screen, allowing the designer to monitor the motion limits of the robot. When the robot attempts to move outside the envelope, the joints involved are flagged on the CRT.

With the cell ready for robot simulation and path generation, the system gives the designer interactive control of each link of the robot by means of menu selection. By using the menu, the designer has many ways of moving the robot about in the workcell. Among the choices are explicit coordinates that use the CAD geometry and surfaces to control the end effector and the use of vectors and incremental distances.

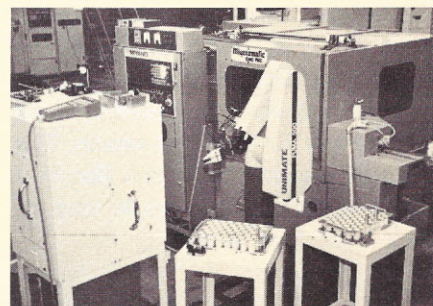
When the "TEACH MODE" is selected from the menu, the image control unit at the CAD terminal becomes a teach pendant. Using this mode, the designer can manipulate each robot link separately in either world or local coordinate systems and can test many end effector positions before teaching the appropriate one as part of the robot path. Other menu options allow the designer to change robot travel speeds, change the working tolerance of the end effector, output a plot of the robot in a particular position, grasp or release objects in the workcell, and add robot language statements at any time during the robot path creation.

Interference checking between the end effector and workcell equipment is carried out by visually following the simulation of the robot during path generation. This may require simulation of the workcell in a number of different perspectives such as the top, front, and side views. Only one view can be simulated at a time.

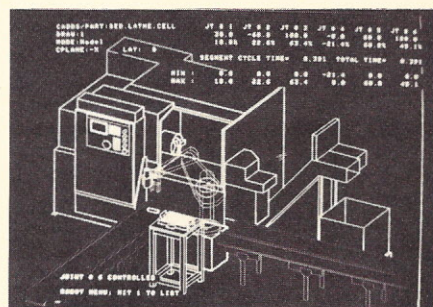
When the designer has finished with the INS ROBOT mode, the robot graphics disappear from the CRT screen, leaving behind the robot path in the workcell. The path appears as a series of lines representing the end effector's motions. The "DISPLAY ROBOT" command simulates the robot in its workcell by either



P1



P2



P3

A Unimate Series 500 robot arm has been programmed to pick up a workpiece and put it into a lathe. When the lathe has finished cutting the part into the desired shape, the robot removes the piece and inserts it into a laser inspection station to be checked for tolerance. The part is then placed into either an accept or a reject pallet. Photo 1 shows the arm, its end effector, and a pallet. In Photo 2, the teach pendant rests on the laser inspection station in the left foreground. Photo 3 is a CAD simulation of the robotic workcell, control station, pallet, and parts conveyor.

dynamically or statically following the selected path. The designer can also edit the path by adding, deleting, or adjusting the robot steps. During path simulation, time cycles are displayed on the CRT screen to allow time comparisons between different robot types.

A satisfactory robotic simulation in the workcell signifies the completion of the initial workcell design. Next comes the detailed mechanical design of special fixturing, end effectors, end-of-arm tooling, and interfacing hardware required in the workcell. Once the workcell design is determined, the robot program can be generated from the simulation path. If necessary, additional paths can be created and verified through robot simulation and used for program generation. To translate the path into the desired robot language, the designer activates the program generator, supplies answers to the CAD system's prompts, and selects the robot path. At this point, the designer can add additional robot language statements to the path. The program generator then translates the path information into the language of the robot to be used in the workcell. The robot language, which is in the form of a text file on the CAD system, can be edited and modified before being transferred to the robot controller by way of either a direct RS-232 link or a modem interface.

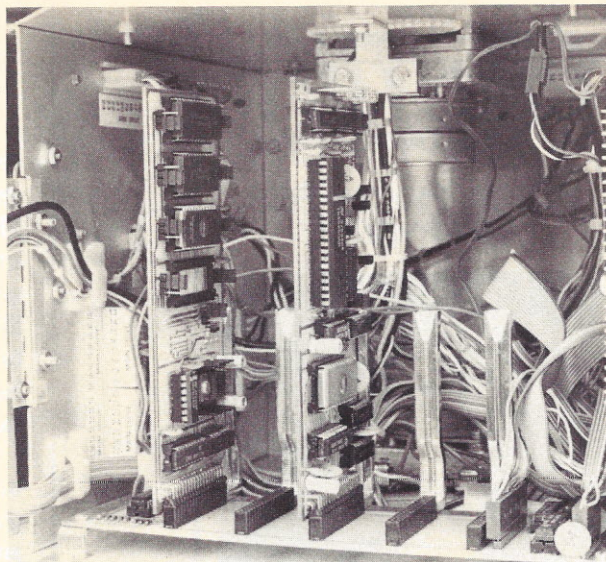
The use of interactive graphics simulation and programming has greatly enhanced the design and production of robotic workcells. Dynamic robot simulation allows the designer to verify the concept's viability from the beginning of the design process. The use of CAD to enhance design efforts, to simulate, and to graphically program the robotic workcell has moved automation closer to a full integration of CAD with CAM.

Robert Boren is a senior engineer in the systems engineering department of Unimation Incorporated.

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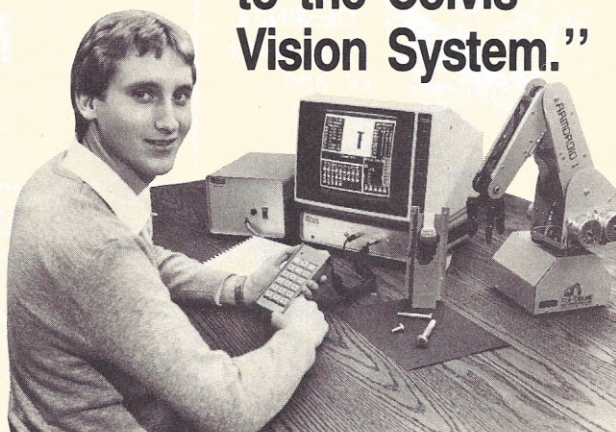
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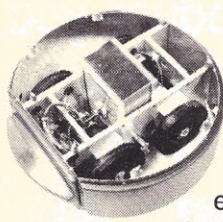
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In The Robotics Age

Continued from page 5

operator what it *thinks* it heard. The user logs on by repeating a series of words spoken by the system, and Intel expects that this "voice print" will be used to keep unauthorized personnel from changing robot control programs.

Two autonomous robots were ensconced in adjacent quarters, the Arctec Gemini and its burly cousin the Denning Sentry. The pair, unfortunately, were rarely allowed out because of the aisle-blocking pedestrian traffic jams that quickly collected. Such clever cylinders, roaming about all on their own, are guaranteed to arouse the Gee Whiz in everyone.

AKR Robotics brought along a muscular prototype 7-axis arm (try to solve for the kinematics on *that* one—and an eighth axis can be added by mounting the arm on a track). The C4000 (Photo 8) aims for

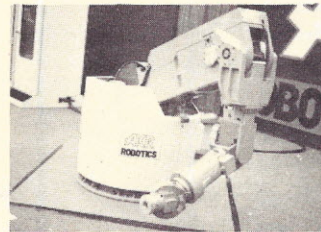


Photo 8.

swiftness and accuracy via brushless electronically commutated motors that produce 2 meters/sec. arm speeds with an acceleration of 10 meters/sec.² carrying a 22 lb. payload. AKR expects the production model to find dispensing applications in the automotive industry.

At Robomatix, we were intrigued by a "Doverential" and a new robot teaching device. The Doverential, named for its designer, is a differential that is belt-driven by two equal external motors (Photo 9). It was

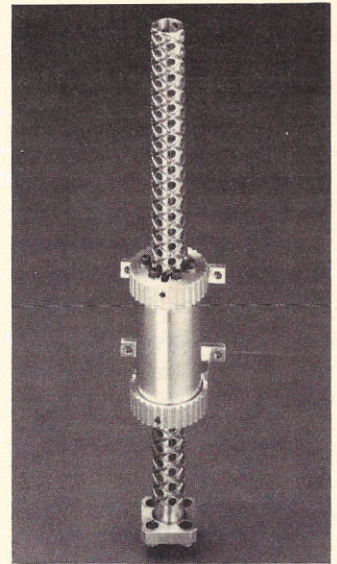


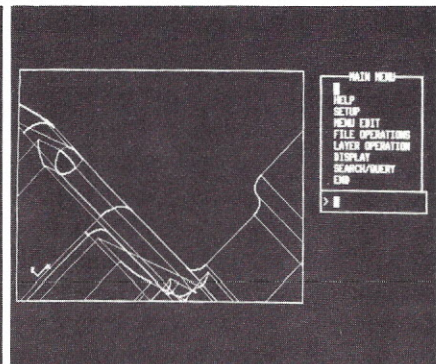
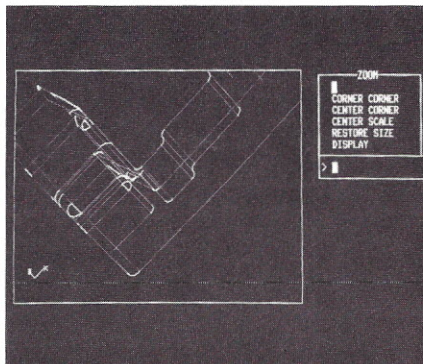
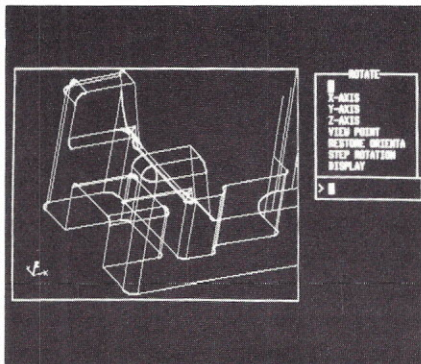
Photo 9.

developed to power the company's direct drive pick-and-place robots and the company expects it to replace conventional one motor-one axis drives by allowing power to be divided between the two motors as required. Robomatix's other unveiling, the Point Teaching Device, allows a trainer to stay out of the robot's work envelope while using a mouse and gridboard to put a SCARA through its paces either by jumps or in a continuous path.

Karel's debut was especially impressive even though Karel could not be seen, for Karel is a new programming language developed by GMF Robotics. The company fully expects Karel to become the industry standard programming language and intends to carry out all its own future programming in Karel. A new vision system has been developed to run on Karel: "In five years," said GMF president and CEO Eric Mittelstadt, "if it doesn't have vision you won't call it a robot." More about Karel will appear in future issues of *Robotics Age*. ■

New Products

IGES-Based Software Interface



Baseview™, a software application tool that allows access to CAD drawing information for display on low-cost terminals is available from Digital Equipment Corporation. Baseview supports the Initial Graphics Exchange Specification (IGES) and can access data from an IGES file that has been created on a CAD system. With Baseview, engineers can electronically

review and approve designs, manufacturing departments can verify parts, and purchasing departments can check part dimensions of a design to facilitate ordering.

Baseview supports a number of graphics terminals, such as Digital's VT240, 241, and 125 units, as well as Tektronix 4014 terminals. The display functions support 3-D drawings and allow users to pan, zoom, and

rotate a drawing. The search and query features let users search for parts of a model and display the dimensions associated with the various elements of the object. For more information, contact: Reggie Redford, Digital Equipment Corporation, Maynard, MA 01754-2198, telephone (617) 467-4035.

Circle 40

New Version of FactoryLink

A new, real-time, multitasking version of FactoryLink™, an industrial information management software system that runs on the IBM industrial PC, has been introduced by United States Data Corporation. Without user programming, the system provides real-time color graphics including process simulation, machine simulation, PID displays, alarms and alarm management, and trending, as well as data logging, file storage, computation, and communications between all types of monitoring and control equipment.

Compatible with other industry software, FactoryLink enables the industrial manager to use such programs as Lotus 1-2-3, VisiCalc, and Symphony on the factory floor to process and manage production and process control information. Its multitasking capability allows the user to run and view more than one program at a time, switch quickly from program to program, and exchange data among different programs.

For more information, contact: Bob Midyett, President, United States Data Corporation, 1551 Glenville Dr., PO Box 850058, Richardson, TX 75085, telephone (214) 680-9700.

Circle 41

CAD for the Personal Computer

The Analyst Toolkit®, compatible with the IBM PC, XT, and AT systems, enables the systems analyst to create dataflow diagrams, entity relationship diagrams, state transition diagrams, and free-form graphics on a unique PC-based environment. The mouse-driven graphics allow the user to create symbolic graphic models on real-time, on-line, and batch processing systems. The Data Dictionary feature allows for the integration of graphic models through the use of the Diagram Extraction facility and a PC-based database manager.

The system can automatically interface graphic models with other development tools such as mainframe data dictionaries and fourth generation languages. The error-checking facilities of the Toolkit ensure the consistency and accuracy of the graphic models. Additional features include a compose facility that allows users to combine graphics and text, providing flexible page formatting and indexing to give documents a finished look.

For more information, contact: Paul Rad-ding, Yourdon Software Engineering Co., 1501 Broadway, New York, NY 10036, telephone (212) 391-2828.

Circle 42

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Circle 12

New Products

Dial Set Capability for CAD/CAM Graphics Systems

Disc Instruments has announced its "Dial Set" capability for large main-frame CAD/CAM graphic systems. This lightweight, low-cost, detachable device contains a fully digitized microprocessor-controlled terminal that can be used in conjunction with Disc's series T500, 3-axis trackball. The T500 provides systems with an X,Y, and user definable third axis that can be used for the rotation of 3-D figures as well as for zoom.

The "Dial Set" capability is a multimodality manual input device that is available in blocks of 4, 8, 12, and 16. It can be purchased in module form for mounting to a user's panel or as a freestanding device designed to the user's specifications.

This advanced graphics capability allows users to vary all perimeters of a 3-D graphics drawing. For example, in an aeronautics application, a user may be working with an airplane design that is rotating in different positions. To separately perform such functions as opening cargo doors, or moving ailerons, rudders, and landing gears into different positions such as up and down or open and closed, the user would simply turn a dial.

For more information, contact: Peter A. Lajoie, V.P. Marketing, Honeywell, Disc Instruments Subsidiary, 102 East Baker St., Costa Mesa, CA 92626, telephone (714) 979-5300.

Circle 43

Pen Plotter for CADstation

The CADstation™ 2/50, a standalone, 2-D design/drafting system tailored for CADAM® users, now supports a pen plotter. The plotter capability allows users with remote sites to obtain a plot directly from the CADstation without accessing the host computer. Initially, the CADstation will support the Houston Instruments DMP-52 plotter, which prints up to D, A1, or architectural sizes at 16 ips axial, 22 ips in all directions, with acceleration up to 4G.

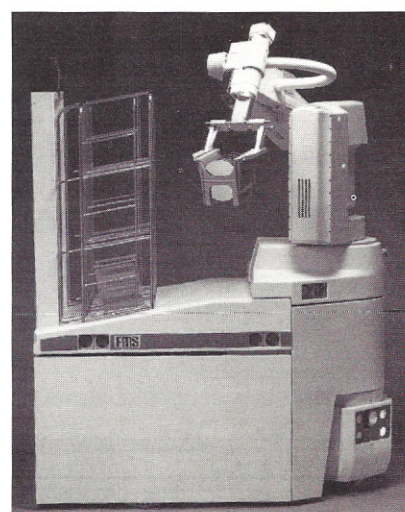
For more information, contact: Linda Popky, Adage, Inc., One Fortune Dr., Billerica, MA 01821, telephone (617) 667-7070.

Circle 45

Computer Integrated Manufacturing System

Flexible Manufacturing Systems has introduced a computer integrated manufacturing system for robotic transfer, storage, and inventory control of semiconductor wafers in clean room manufacturing. The Model 4020 CIM System, which is aimed at reducing particulate contamination and misprocessing of wafers, is expected to improve fabrication line yield by 20 percent. Computerized routing of work in process, based on wafer lot status and equipment availability, will shorten process cycle time and reduce work in process inventory, the company says.

A basic CIM System will service 10 workstations. It includes a mobile transport vehicle that is self-contained and self-propelled, two intelligent work in process stations that provide local storage and inventory control of wafer cassettes or boxes, 10 docking modules and operator interfaces, and one central control computer with an uninter-



ruptable power supply, a graphics terminal, and software.

For more information, contact: Joseph A. Nava, Vice President, Marketing, Flexible Manufacturing Systems, Inc. 16780 Lark Ave., Los Gatos, CA 95030-2315, telephone (408) 395-2777.

Circle 44

AUTOCAD Version 2.1

A new version of Autodesk, Inc.'s AutoCAD™ program, a multipurpose 2-D drafting and 3-D visualization software package for engineers, designers, and drafters, has been released. Improvements include an interactive pick function, the elimination of limitations on drawing complexity, and a variable and macro facility to provide users and application developers with the tools to create custom commands and functions.

AutoCAD's new visualization capability, included in Advanced Drafting Extensions 3 (ADE 3), is the first step in moving AutoCAD toward a full-capability 3-D design tool. Wire-frame drawings can be viewed and plotted, and lines that are obscured by other objects in a 3-D drawing can be visually suppressed from displays and plots. ADE 3 also contains the ability to create and edit polylines.

AutoCAD users can expect an overall 40 percent improvement in the program's execution speed over AutoCAD Version 2.0, the company says. The plotting of drawings has been optimized as well, and AutoCAD

now supports many dot matrix and laser printers.

For more information, contact: Sandra Boulton, Autodesk, Inc., 2320 Marinship Way, Sausalito, CA 94965, telephone (415) 332-2344.

Circle 46

Quick Change Tooling Adaptor

The EOA D-TRAN 25 Quick Change Adaptor, part of the company's robotic end-of-arm tooling line, automatically quick disconnects air, fluid, vacuum, and electrical connections for the D-TRAN Multi-Grip family of grippers. The short adaptor (1.5 in. long) minimizes tool point deflection, weighs less than 1 pound, and carries a 25-pound payload. Position accuracy of 0.001 in. meets operating specifications for Seiko D-TRAN robots.

For more information, contact: Keith Crawford, EOA Systems, Inc., 15054 Beltway Dr., Dallas, TX 75234, telephone (214) 458-9886.

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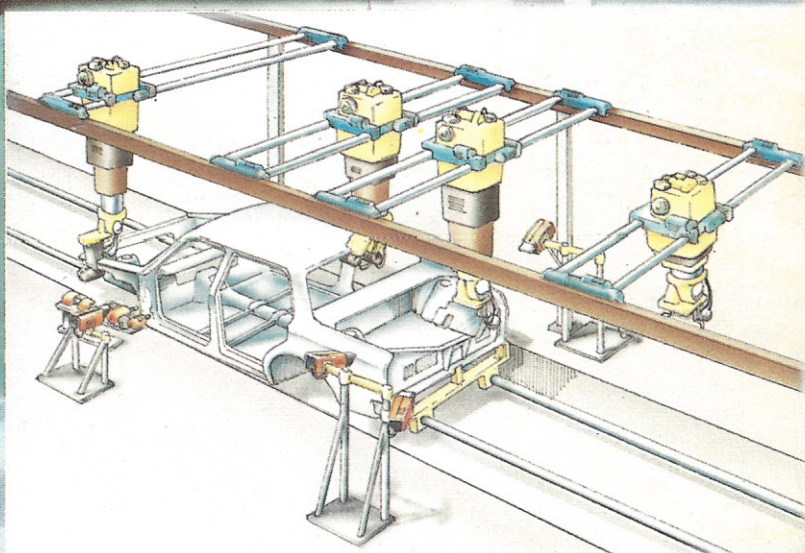
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